times towards the end of the book. But those items are more interesting than annoying.

Of course I second Trimble's recommendation: "Please read the book." And read her review. — Phillip Helbig.

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An Introduction to General Relativity and Cosmology, 2nd Edition, by Jerzy Plebański and Andrzej Krasiński (Cambridge University Press), 2024. Pp. 577, 17·5 × 24 cm. Price £69·99 (hardbound; 978 1 00 941562 0).

Both authors are well known for their highly mathematical approach to General Relativity (GR), which had a strong tradition in the former Soviet Union (Sakharov, Zel'dovich, Novikov, *et al.*) and many neighbouring countries (both authors are Polish, though the first author spent a substantial fraction of his life in Mexico). Some might quibble with the title; of the many books I've read covering both GR and cosmology, this book is both one of the longest and the most mathematical. The second author (the first died in 2005) is aware of the tension between the title and the contents, mentioning it in the preface to this second edition, and justifies calling it an 'introduction' because not all topics are covered* and because no prior knowledge of GR or differential geometry is assumed, though knowledge of calculus, Special Relativity, classical mechanics, and electrodynamics is assumed (thus one could start learning GR and cosmology with this book, though the author notes that "[it] takes a careful reader to some height of advancement"). This is very much a 'maths first' book which, despite the author's caveat, covers a large range of topics; that it also does so to a significant depth while 'showing much of the work' explains the length. The first part of eleven chapters (at only a bit more than a hundred pages) covers 'Elements of differential geometry' while the second, with thirteen chapters (but about four-hundred pages), 'The theory of gravitation'. The first part is rather standard, though it does mention Bianchi models and the Petrov classification (though that chapter, like several sections, is marked with an asterisk as being less relevant and more advanced, sort of like 'track two' in *MTW*[1]). The second part includes chapters on standard topics such as the Einstein equations, relativistic cosmology, and the Kerr and Schwarzschild

*Missing topics which are mentioned are gravitational waves, the Cauchy problem, generating new stationary-axisymmetric solutions out of known solutions, the Penrose transform, cosmic censorship, experimental tests, spinor methods, relativistic astrophysics, history of relativity, and Special Relativity.

metrics, but also topics which obviously reflect the interests of the authors, such as the Kaluza–Klein theory, Lemaître–Tolman[–Bondi] models, and Szekeres geometries; a short chapter on relativistic hydrodynamics and thermodynamics and one on the Global Positioning System are more of an attempt to include at least a brief overview of topics which are obviously important in a practical context or currently hot topics*, as opposed to more specialized topics, many of which are covered in some detail.

So it doesn't cover everything. However, it does cover a lot of ground, though of course it is necessarily restricted in the discussion of the various individual topics, about many of which books of similar length have been written. So what is the attraction of a book which covers several topics in a fair amount of detail, as opposed to a *really* introductory book then additional in-depth books for more specific topics? One possibility is that it is a good book if one wants to learn GR in some detail with applications to many fields presented in a uniform notation (different notation schemes, especially regarding signs, are a constant concern when studying GR); apart from worked examples in the main text, there are exercises at the end of most chapters (no solutions, but the last chapter is entitled 'Comments to selected exercises and calculations'). Another is that it is very well written, perhaps surprising since neither the second nor (as far as I know) the first author is a native speaker of English. (Krasiński mentions on his website that his only native language is Polish. I strongly doubt that Plebański was a native speaker of English. However, I know of an astronomer from a non-English-speaking country with a name typical for that country and who grew up there who nevertheless is a native speaker of English as well.) Indeed, the language is better than in many books written by native speakers: there are few typos, and I even have fewer complaints about style than I normally do when reading a book. Other useful features are eighteen pages of somewhat smaller-print references, including titles and the page(s) on which each is cited in the text, and a thirteen-page index (in the usual small print often used for indices). I also enjoyed the footnotes, which are often comments on the history of the topic. Occasionally, there are such remarks in the main text, or gems such as the description of the Bergmann–Wagoner theory: "... a curiosity because it is far from being well understood". From others, it is obvious that the authors are very familiar with the literature: "But this is where most textbooks make a mistake...."; "This second condition was found by Hellaby and Lake (1984), but in their paper it is hidden as two humble numbers in tables and a one-line comment and seems to have been overlooked by all later authors." There are a few black-and-white figures scattered throughout the book; except for two pictures of gravitational-lens systems, they are diagrams of the sort one expects in such books.

Some things were also a bit surprising. As mentioned, sign conventions always need to be kept in mind when studying GR, but I don't think I've ever come across Λ accelerating the expansion of the Universe when negative; when

*For example, a huge amount of work involving numerical relativistic hydrodynamical simulations has been done in order to interpret what is seen by the *Event Horizon Telescope*.

discussing cosmology in more detail, though, "[f]ollowing Friedmann we denote $\Lambda = -\lambda$".* (Note that these days, usually λ is the 'dimensionless cosmological constant' equal to $\Lambda/(3H^2)$.) Even apart from my own interest in the flatness problem, the discussion here certainly deserves special mention, starting out with a warning that "The views expressed in this section are A. K.'s. J. P. bears no responsibility for them." I basically agree with his discussion of the flatness problem itself, but instead of considering arguments claiming that it is not really a problem even within the context of the Friedmann models[6], he points out that it is "completely transformed if we consider the Lemaître–Tolman (L–T) and Szekeres models" — while that is true, it is probably irrelevant to our Universe.

Electrically charged black holes (Kerr–Newman if they are spinning, Reissner–Nordström if not) often get short shrift because they are thought to be rare. This book, though, has a fair amount of discussion on them, highlighting many interesting and unexpected (at least for me) aspects. That is true in general: although Lemaître–Tolman[–Bondi] and Szekeres models are more general than the Friedmann models usually used in discussing cosmology, it seems doubtful that they apply to our Universe, but they are discussed in great detail (not only within the context of the flatness problem as mentioned above). (To be sure, the second author has used them to try to explain the acceleration without dark energy, but I'm sure that they would have been included even if the Universe were not believed to be accelerating.) Of course, there are other topics once thought to be interesting but irrelevant — an example is redshift drift (sect. 17.10); however, due to advances in technology it can now be studied in detail[7].

Apart from the claim that one needs to know H_0 in order to measure q_0 from the magnitude–redshift relation†, I noticed no real mistakes, at least not if we can forgive the authors (both Polish, the second associated with the Copernicus Astronomical Center) for claiming that "Copernicus was the first astronomer who noted that the Earth is not at the centre of the Universe". (Copernicus is introduced in connection with the Copernican Principle that we are not located at a special place in the Universe.) However, I do think that their claim "that virtually the whole of observational cosmology is based on

*At first I thought that it was a typo rather than an unusual sign convention. Almost 30 years ago I corresponded with the late Steven Weinberg regarding a sign error in his famous textbook[2] which covers ground similar to the one reviewed here; that also involved an unexpected minus sign accompanying the cosmological constant. I sent him an email after I had convinced myself that it was actually inconsistent and not some unusual convention. We eventually found out that it was an actual typo in some printings of his book. I was surprised that he invested so much time tracking down a typo in a book written decades earlier. A few weeks ago, while listening to a seminar talk I learned that there is a more serious error in that same book, which is due to the propagation of a typo from Messiah's textbook[3]; undoubtedly many have also quoted Weinberg's expression without noticing the typo. I'm sure that that explains his dedication and attention to detail. Many years later, I reviewed[4] another[5] of his books and sent him a list of minor mistakes. Again, I was surprised about how concerned he was with them.

†While important historically[8], observational cosmology has moved beyond trying to measure only H_0 and q_0. The latter is the first non-linear term in a Taylor expansion, and thus was important when redshifts were small and distance calculation for general Friedmann models was difficult; neither is the case today.

the Friedmann–Lemaître models is a consequence of inertia in thinking and of emotional attachment to the doctrine of equivalence of all positions in the Universe" is exaggerated. Certainly a hundred years ago simple models were assumed because, with practically no data, they were as good an approximation as any and calculations are easier in them. But even before the first edition was written (2006), the idea that the Universe is homogeneous on large scales had become an observational fact (see the discussion in a book[9] by an expert in the field reviewed in these pages[10] a few years ago). Probably related to that is a sceptical attitude towards the standard ΛCDM model of the Universe and the hope of the authors that alternative explanations for the claim that acceleration has been observed might prove to be right. In another context, the authors note that one of their ideas (an attempt to explain gamma-ray bursts *via* blueshifted radiation from a non-standard Big Bang) has "met a violent opposition from astrophysically minded referees and will likely not be further pursued". There is a good discussion of the definitions of cosmological distances, but I was somewhat surprised that the simplest generalization to a more realistic universe[11,12] is not mentioned, though more complicated effects such as the position drift of light sources (due to moving matter sweeping along light rays passing through it) are.

This book has a very different balance among the various topics than that of otherwise broadly similar books. More detailed discussion of those related to our Universe can be found elsewhere, but this book is the place to go for interesting if not necessarily relevant details which are hard to find elsewhere, in addition to those reasons mentioned in the second paragraph above. After I had written this review, I came across a link[13] on Krasiński's personal web page to the review in this *Magazine* by Alan Heavens of the first edition[14]. His review is rather similar, but as expected shorter than mine. I can't improve on his recommendation: "For anyone looking for a thorough mathematical treatment of General Relativity, or for a supplement to existing books, this is highly recommended. It is not a standard text by any means, but I would be surprised if there was anyone who didn't find in it something new, interesting, and enlightening". — Phillip Helbig.

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