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REFRACTION NEAR THE HORIZON — AN EMPIRICAL APPROACH PART 2: VARIABILITY OF ASTRONOMICAL REFRACTION AT LOW POSITIVE ALTITUDE (LPAAR)

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The first part in the series on 'Refraction near the horizon' treated the terrestrial refraction of the Dip[1,2]. This second part investigates the diurnal and seasonal variabilities of LPAARs at coastal locations in temperate climates. The refractions are calculated by ray-tracing using typical temperature-gradient profiles from statistical analyses of radiosonde data. The theoretically-calculated refractions are compared with a re-analysis of the work of Argelander and Bessel, as well as with new observations. The results compare well with measurements made at night or during sunrise/sunset. The selected procedure provides also estimates for daytime LPAARs which do not appear to have been measured before. Differences between published LPAARs can be explained when relating them to their corresponding part of the day. For converting LPAARs to different atmospheric conditions it was found that this depends in addition to temperature and pressure also on the time of the day, on the height of eye (*HoE*) above the Earth's surface, and on the viewed altitude. For a given part of the day and *HoE*, reasonably accurate conversions were obtained with higher-order polynomials as a function of temperature and pressure, and of the viewed altitude. It is found that for an observer near sea level the refraction's diurnal cycle varies at the astronomical horizon by about 6 arcmin with a maximum near sunrise and a minimum shortly after solar transit; refraction changes faster after sunrise than after sunset. The seasonal LPAAR variations are found to be less than about ±0·7 arcmin when freed from the seasonal temperature and pressure variations. Daytime refractions indicate a systematic seasonal pattern with maximum refractions during mid-Winter and minimal refractions during late Spring, having at $Alt = 0°$ a maximum range of 1·3 arcmin. Variabilities beyond the 'seasonal' ones are attributed to 'local' variabilities, having at $Alt = 0°$ a maximum contribution of about ±0·5 arcmin. At $Alt = 0°$ variabilities between individually observed LPAARs are (without dip contributions) maximum near sunrise (±4·6 arcmin) and minimum during daytime (±1·8 arcmin). The non-linear temperature-gradient profiles near the surface during the day, the night, and near sunrise make the refractions more sensitive to *HoE* differences than during sunset.

1. Introduction

As early as around 1750 it was found[3] that the astronomical refraction over a wide range of zenith distances can be well described with some approximations using the tangent function and only depending on the atmospheric conditions at the observer (Oriani's theorem[4,5]). It was, however, soon noticed that those approximations deviate noticeably when the zenith distance exceeds about 85 degrees. Bradley started to measure low-altitude refractions around 1750 at the Royal Observatory at Greenwich that were published after his death by Maskelyne[6] and Bessel[7]. Argelander made such measurements for Bessel in 1820/21 at the Königsberg observatory[8]. In 1857, Rev. Main proposed improvements to Bessel's values[9], based on his own observations made at the Royal Observatory and at the Observatory at the Cape of Good Hope. Their observations showed that refractions at low altitudes depend differently on atmospheric temperature and pressure than at higher altitudes where the dependence of astronomical refraction on pressure and temperature was found to correspond to that of the refractivity of air itself. It was thus found that the refractions near the horizon do not only depend on the atmospheric conditions at the observer but also on the 'weather', *i.e.*, on the atmosphere's density distribution. Young[4] estimates that the transition from where the astronomical refraction depends primarily on the atmospheric conditions at the observer to where atmospheric structures become important is between altitudes of about 2 to 3 degrees. Schmeidler[10] gives a brief account on the first findings related to the diurnal and seasonal variations of astronomical refraction, which — according to him — were first observed in the second half of the 19th Century at the Pulkovo observatory. Later, in the 20th Century, researchers determined low-altitude refractions and their variability mostly from sunrise and sunset observations[11–14] or by numerical calculations using some atmospheric model[15–18].

Ray-tracing can calculate the refraction for any horizontally stratified atmospheric model, and it has been shown[17,18] that even simple models based on the standard atmosphere give results that agree well with 'average' refractions. However, in order to describe diurnal or seasonal variations requires atmospheric profiles which describe in more-realistic models the different 'typical' atmospheric conditions.

Diurnal and seasonal variations of atmospheric refraction have also been intensively studied in relation to *terrestrial* refraction by surveyors and navigators (dip of horizon). Bauernfeind[19] and Thom[20], both being surveyors, have probably provided the most-detailed results on the variability of the terrestrial refraction. I am not aware that the variability of the *astronomical* refraction has before been analysed in a similar, systematic way, covering all parts of the day and of all seasons. This study aims at filling some of this gap and may thus become useful for astronomers and archaeoastronomers[21], and for traditional navigators and surveyors.

Possibly the first account of diurnal refraction variations were those observed by Perrault[22] in 1674. Later, in 1749, Bouguer[23] made one of the first quantitative assessments of the refraction's diurnal variability with his rule that during the day refractions would be ⅙ to ⅐ lower than at night. One of the more recent systematic attempts to quantify diurnal differences of low-altitude astronomical refractions appears to have been the sunrise/sunset observations by Hellerich[11], by Seidelmann (published in Schaeffer & Liller[24]), and by Sampson *et al.*[13]. Note that the sunrise/sunset observations performed by Schaeffer, Liller, and others will not be discussed in this study on LPAARs because their observations were made exclusively at (large) negative altitudes which the author intends to treat separately as another part of this series on refraction near the horizon.

Sugawa[25] and Sampson[12] investigated also monthly refraction variations. Sugawa derived them in a similar way as is done here, *i.e.*, by ray-tracing calculations using observed temperature-gradient profiles from night-time radio-soundings made in Sendai, Japan. Sampson shows monthly refraction variations from sunset and sunrise observations at Edmonton, Alberta. It is understood that both related their refractions to the actual temperatures and pressures at the time of the observations; the variability of their refractions thus reflect the much larger effect of the varying seasonal mean temperatures. Here the refraction's seasonal variability contribution is determined independent of seasonal temperature and pressure variations.

This study determines astronomical refractions and their variations for all parts of the day and for all seasons by making use of the world-wide available radiosonde data, and then by determining the astronomical refraction indirectly by ray-tracing calculations on statistically derived typical atmospheric models where the density variations are obtained from temperature-gradient profiles. For verification, the resulting temperature gradients in the lowest layer are compared with directly measured ones. Using these atmospheric models the refractions are then calculated at standard conditions and the results compared with published values from Sun and star observations. Comparing different results requires converting the refractions between different atmospheric conditions. At low altitudes this conversion is more complex than at higher altitudes, because it depends not only on the meteorological parameters (temperature and pressure at certain *HoE*) but also on the atmosphere's temperature-gradient profile, and on the viewed altitude. If the conversion has to be accurate over a wider parameter range then the correction functions require higher-order polynomials.

2. Statistical derivation of typical temperature gradient profiles

Radiosondes are launched worldwide at 00 UT and 12 UT, occasionally also at 06 UT and 18 UT. Depending on the stations' locations and the season, the launch occurs at a different part of the day. A statistical analysis of all temperature-gradient profiles derived from soundings made within the same season and within the same local hour, *LHr* (as defined in the next section), is expected to reflect the typical features for the corresponding atmospheric condition. For this study 27 sounding stations distributed around the globe have been selected which are all near a coast and in temperate climates (Köppen Class C[26]). The sounding data from these stations have been retrieved from the IGRA[27] archive. Before the temperature gradients from the sounding data can be statistically analysed, the content of the IGRA files require some conditioning. With the help of radiosonde metadata[28], soundings are selected only if the type of temperature sensor and its time constant are reasonably well known[29], typically within one second. The preparatory steps determine the *LHr* and the temperature gradients from the temperature and geometrical height differences, the latter corrected with the time constant of the *T*-sensor as proposed by Jensen[30] and further improved by Mahesh e*t al.*[31]; this correction thus disregards all measurements earlier than two time constants after launch, and moves all temperatures downwards by one time constant assuming an ascent rate of 5 m s^{-1}.

After the preparatory steps, the typical temperature-gradient profiles are found from a statistical analysis of all available sounding data. For this analysis the temperature gradients (as they contribute for a station at one of the seasons to one of its local hours) are saved in statistical bins representing geometrical height increments (layers) of an atmospheric model above the station, having increment sizes of 20, 50, 100, 250, and 500 m within the height ranges 0–1000 m, 1–2·5 km, 2·5–5 km, 5–10 km, and 10–20 km, respectively. The resolution of a 20-m-high temperature gradient next to the surface may appear coarse for the intended purpose, but a finer resolution would then likely also require distinguishing between different surface types. Note that in the figures the values for a layer are always shown at the bottom. Above 20 km, all temperature-gradient profiles contain the same temperature gradients as they are defined by the temperatures at the corresponding geometrical heights of the standard atmosphere.

For the statistical analysis the temperature gradient from the enhanced sounding data is put in each bin of the gradient's height range. As an example: a certain gradient *x* results from the difference of the temperatures at 20 m and at 80 m. Now, all bins within this height range, *i.e.*, the bins for 20–40 m, 40–60 m, and 60–80 m altitude receive each the value *x* for subsequent analysis. A filter removes from the statistics extreme temperature gradients which are expectedly erroneous values. The number of removed gradients depends on the station; maximum values are a few per thousand. Temperature gradients are also discarded when the station pressure is below the lower standard deviation from all selected soundings of the particular station within the corresponding season; that considers in some way that under very low air pressure an object outside the atmosphere would likely not have been visible and that the sounding is not expected to be representative for the condition of an astronomical observation. The number of accepted soundings entering the statistical bins varies from none up to a few thousands but only temperature-gradient profiles derived from at least ten soundings are used for subsequent analyses. After passing all the soundings of a station, the mean, median, and an estimated mode are calculated for the content in each bin, but using for this study only the mean. When analysing a sounding performed at a location *l* at a certain date and time its gradients probably have some typical characteristics for the corresponding part of the day (local hour *h* or *LHr*), and the corresponding annual season *s*. A set of mean values in each of these height-related statistical bins describe therefore one *basic temperature-gradient profile* where *l*, *s*, and *h* are identical; such a profile is designated as T'_{lsh} or as T'_o and its refractions calculated by ray-tracing as R_{lsh}

or as R_o.

These basic temperature-gradient profiles provide different options for calculating averaged refraction results. One can, *e.g.*, calculate first an 'average' profile from a set of selected temperature-gradient profiles and determine then with one ray-tracing calculation its 'average' refraction. Alternatively one can also calculate the refractions for each temperature-gradient profile and calculate then the 'average' from all these refraction results. Where not explicitly stated differently, the average refraction is calculated with the mean temperature-gradient profile and their variability with the refractions from each temperature-gradient profile either from all the results within a local hour, within a season, or from all seasons.

3. Hourly and seasonal resolution

The local hour, *LHr*, is related to sunrise ($LHr = 06$), Sun transit ($LHr = 12$), and sunset ($LHr = 18$). It was found that the diurnal results could be grouped into four cardinal parts of the day as shown in Table I. The refractions between $LHr = 23$ and $LHr = 7$ of the next day were found to vary little, and could be grouped to represent the night-time results; similarly the daytime results were grouped with data from $9 \leq LHr \leq 17$. Sunrise and sunset contain data from $5 \leq LHr \leq 7$ and $17 \leq LHr \leq 19$. With the intention of possibly finding a better cyclic description of the seasonal variation, the year has been divided into the six seasons as defined in Table II. The symbols in the two tables are those used to show their data in the various figures.

TABLE I

The four cardinal parts of the day

Cardinal part	*LHr:* from	to	Symbol
NIGHT	23	7 +24	
SunRISE	5	7	
DAY	9	17	
SunSET	17	19	

Note that the night-time includes also the sunrise whereas the daytime does not include sunset.

TABLE II

Definition of the six annual season

5_wAut:	Oct 15 – Dec 14	Late Autumn	X
0_Wint:	Dec 15 – Feb 14	High Winter	O
1_wSpr:	Feb 15 – Apr 14	Early Spring	+
2_sSpr:	Apr 15 – Jun 14	Late Spring	+
3_Summ:	Jun 15 – Aug 14	High Summer	O
4_sAut:	Aug 15 – Oct 14	Early Autumn	X
9_ALL:	all seasons		

Half-years correspond to:
Winter half-year: 5, 0, and 1
Summer half-year: 2, 3, and 4

4. The resulting typical temperature-gradient profiles

(*i*) Comparing the lowest temperature gradients with those from direct measurements.

Refractions at low altitudes are mainly governed by the temperature gradients in the lower atmosphere where the temperature gradients close to the Earth's surface tend also to differ most from the normal adiabatic lapse rates. Near the surface the vertical temperature profile is logarithmic; this means that the temperature gradient within a height interval depends also on the height of the interval. Larger height intervals and/or higher heights above the surface thus reduce the temperature gradients, while they are greatest for small layers very close to the surface. For this study it is therefore of some importance that the temperature gradients near the surface resulting from the statistical analysis of the sounding data compare 'reasonably' well with actual, directly measured ones.

Various authors have previously investigated the temperature gradients close to the surface over extended time periods with temperature sensors placed on towers at different heights. Knoch[32] measured at the meteorological institute at Potsdam, Germany, the temperatures in Stevenson screens, one on a meadow at 2 m and the other one on a tower at 34 m above the surface. Johnson[33] measured the temperature differences between 1·2 m and 7·1 m and also between 1·2 m and 17·1 m on a tower placed on the Salisbury Plain near Porton, UK. The data from Knoch and Johnson are from similar temperate climates as the selected sounding stations, except that they are far from large bodies of water, whereas the sounding data used here are all from coastal locations. Johnson's measurements between 1·2 m and 7·1 m are from a much lower layer than those from the sounding data; they have only been added for an additional comparison. Also the tower measurements made by Singer & Raynor[34] on Long Island, USA (at the edge between temperate and cold continental climate), and by Flower[35] at Ismailia, Egypt, in an arid desert-type climate, have only been added for an additional comparison. The results for Long Island have been digitized from their figure showing the temperature gradients for 'average' meteorological conditions.

When comparing the temperature gradients in the lowest layer of the sounding data (0–20 m, corresponding to approx. 1·5–21·5 m above the surface) with those of the tower measurements one has to remember how the sounding results have been obtained. It is first up to the operators of the sounding to decide which of the sonde's measured temperatures are sufficiently significant to be communicated, and this may — in addition to changing weather conditions — also show some occasional differences from one person to another. As a result of this the heights of the first transmitted temperatures, which then are used for calculating the lowest temperature gradient, vary considerably, roughly in a range between about 15 m to 150 m above the station with a mean around 40 m and 50 m (logarithmic distribution). Note that the values of all these first temperature gradients enter the statistical bin 0–20 m, regardless of the gradient's height.

Fig. 1 compares the temperature gradients in the lowest layer of the sounding data, as shown in panel *a*, with those from the tower measurements. The mean temperature gradients of the six seasons (black line) range in average from +16 K/km for the night-time values ($23 \leq LHr \leq 7+24$) to –22 K/km for the daytime ones ($9 \leq LHr \leq 17$). Under (*iii*) below it will be shown that with the logarithmic correction these original values change somewhat to +23 K/km and –27 K/km. The sounding data in panel *a* agree best with those from Potsdam in panel *c*, measured at 2 m and 34 m. This agreement appears to result from both having a similar climate, and from both relating to similar heights, the tower of 34 m compared to the layer height of 20 m containing data of the first sounding measurements at average heights around 40 m to 50 m. The Potsdam station has in its neighbourhood also some surface water from the River Havel which forms there a small lake. The temperature gradients measured near Porton (panel *b*) relate to about half of the height difference at Potsdam (1·2 to 17·9 m *versus* 2 to 34 m), resulting in a temperature-gradient range which is about twice as large; similarly, the lower height at Porton (panel *d*) has a height difference which is about ⅓ of the higher one (1·2 to 7·1 m *versus* 1·2 to 17·9 m) resulting in a temperature-gradient range which is again about twice as large. The Long Island temperature gradients shown in panel *e* are strongly influenced by large bodies of waters, the Atlantic on one side and the Long Island Sound on the other. The height difference of 34 m starts high at 11 m,

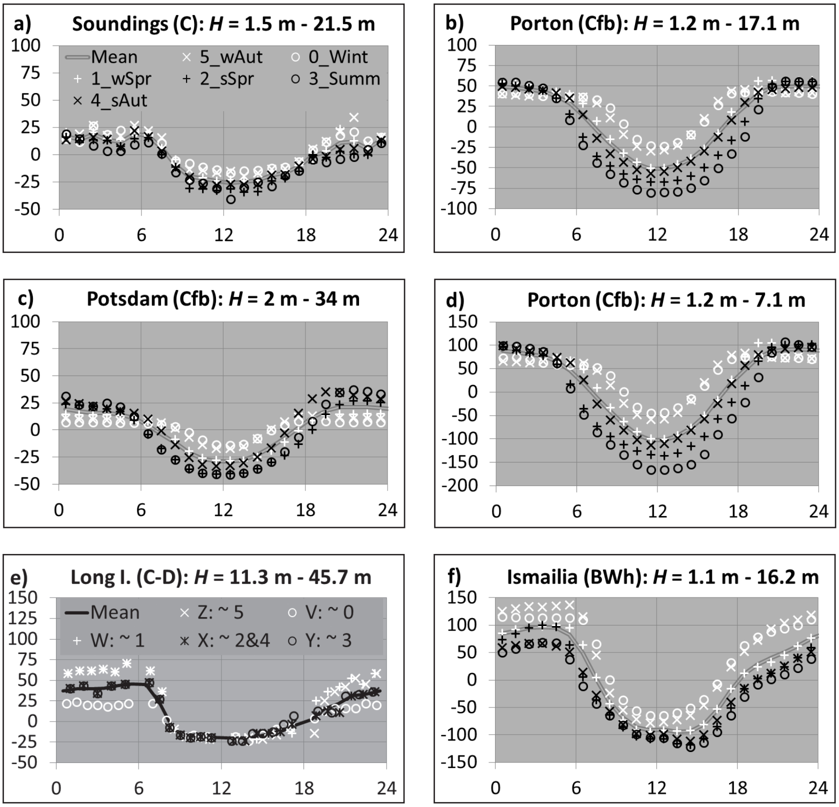


FIG. 1

This figure compares the seasonal near-surface temperature gradients $\Delta T/\Delta H$ [K/km] as a function of local hours. Panel *a* shows those from the analyses of the radio soundings as they result for the lowest statistical bin (0–20m). Panels *b* to *f* show temperature gradients as measured at different locations and between different heights. The various authors published their measurements mostly as mean values for each civil time (0–23 hr) of each month. It is not clear whether they relate to the beginning of the indicated hour, or whether the mean result does indeed relate to the indicated time. Their civil times have been related to local hours (*LHr*) using for each month the mean time for sunrise, meridian transit, and sunset. The monthly results have then been averaged for the corresponding two-month seasons. Due to the various uncertainties the resulting diurnal cycles are expected to be somewhat smeared related to the local hours.

thus at quite some distance over the surface compared to the approx. 1·5 m of the other measurements; they related their data to the five annual seasons 'V' to 'Z'; panel *e* shows how they approximately correspond to the annual seasons '0' to '5' as used here. The city of Ismailia is next to a small lake which is part of the Suez Canal. Due to the strong solar heating, the range of temperature gradients is about twice as large as the one near Porton (panel *b*) which was measured between similar heights.

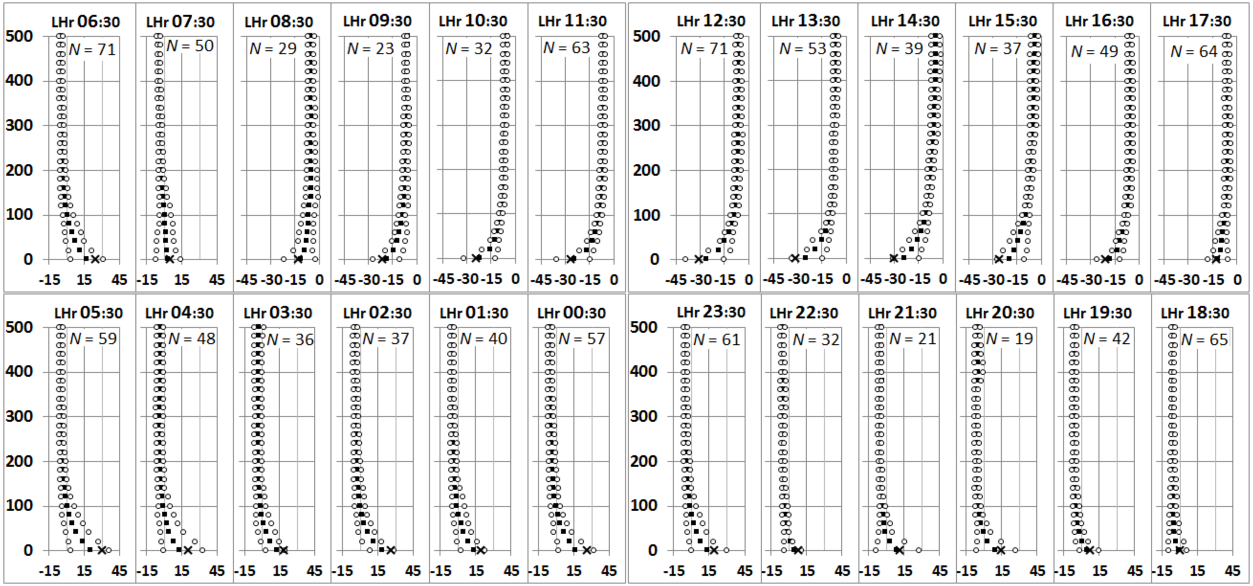


FIG. 2

Mean temperature gradients in K/km (*x* axis) as they result from the statistical analysis for each local hour in the lowest 500 m. The indicated local hour represents the beginning of the one hour interval; the profile is therefore expected to be representative for the conditions half an hour later. The mean values are shown as black squares and the two quantile limits, ⅙ and ⅚, as circles. The value *N* indicates the number of values in the corresponding sets. The x symbol at the lowest layer indicates its logarithmic-corrected mean value. Note that standard atmospheres which model the troposphere with a constant temperature gradient of –6·5 K/km agree best with the conditions near sunset, *i.e.*, between the two panels *LHr* 17:30 and 18:30 but also to those in the morning transition, about two hours after sunrise.

(*ii*) Diurnal and seasonal properties of the temperature gradients from the soundings.

The temperature gradients from the sounding data reflect also the general pattern of the diurnal and seasonal variations. Fig. 2 shows how the hourly mean temperature gradient profiles, T'_h, from the soundings describe the diurnal gradient cycle, starting at sunrise or somewhat before with the maximum inversions, reaching shortly after local noon maximum adiabatic gradients, which then gradually reduce as the Sun approaches the horizon, and turn shortly after sunset again into inversions. This cycle compares reasonably with the Reijs[36] analysis of the *Stability Classes* as derived from three years' observations at the coastal Pilgrim power plant right next to the Plymouth Bay, MA, USA (Köppen C). He finds that the relevant daytime stability classes start significantly increasing at *LHr* 07, reaching a maximum at *LHr* 13, and becoming again minimum near *LHr* 18.

Lapworth[37] observed over several years the diurnal boundary-layer properties at Cardington, UK (Köppen C, far from large water bodies), and finds for this location that (*a*) the morning transition, *i.e.*, when the boundary layer is neutral over the entire layer height occurs — independent of season — about 2 hours after sunrise. In the temperature-gradient profiles of Fig. 2 this corresponds to where all layers have about the same temperature gradient; this occurs between local hours 7:30 and 8:30, thus about two local hours after sunrise which is about in agreement; (*b*) the boundary layer's full convection occurs on average between about 11 and 12 UT, *i.e.*, somewhat before local solar transit, which is at this location on average at 12:02 UT. The analyses of the sounding data do not contain information on the wind. However, if one relates full convection to maximum adiabatic temperature gradient above the surface, then this situation relates in Fig. 2 to the profile at local hour 12:30. The sounding data thus indicate this situation about an hour later; (*c*) the evening transition, *i.e.*, when after full convection and about 2 hours before sunset the boundary layer is again neutral over the entire layer height. In the temperature-gradient profiles of Fig. 2 this occurs between local hours 17:30 and 18:30, thus close to sunset which is about 2 hours later.

It thus appears that for locations far from large bodies of water the maximum adiabatic temperature gradients and the night-time inversions are reached earlier and may therefore have at sunset already small inversions. Some of this difference could also result from Lapworth, removing for quality-control purposes, days from the dataset during which no convection occurred, and also days in which significant advection occurred. The data from this study represent, however, a mean of all meteorological conditions, and contain expectedly also noticeable advection contributions from onshore and offshore winds.

Fig. 3 shows that the mean seasonal temperature gradients, T'_s, from the soundings do also reflect the seasonal variations with the strongest inversions during winter nights and the strongest adiabatic gradients during summer days. The temperature gradients of the night-time sounding reflect also the height increase of the troposphere from about 11 km in winter and early spring to about 15 km in summer and early autumn; this part of the figure is, however, not shown.

(*iii*) Logarithmic temperature distribution near the surface.

Collecting a single temperature gradient in each bin within its height range for the statistical analysis makes the mean temperature gradient in the lowest (0–20 m) layer deviate from the logarithmic shape near the surface. Fig. 4.1 shows the seasonal temperature gradients T'_s as a function of $\Delta\mathrm{Ln}(H)$ =

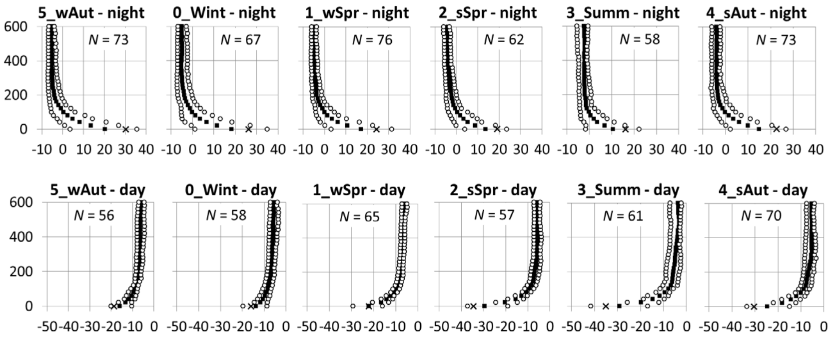


FIG. 3

Mean temperature gradients in K/km (*x* axis) as they result from the statistical analysis for each season in the lowest 600 m for the night times with 23 ≤ *LHr* ≤ 7+24 and for the daytimes with 9 ≤ *LHr* ≤ 17. The value for *N* indicates the number of data for calculating the mean (black squares) and the two quantile limits at ⅙ and ⅕ (circles). The x symbol at the lowest layer indicates its logarithmic-corrected mean value.

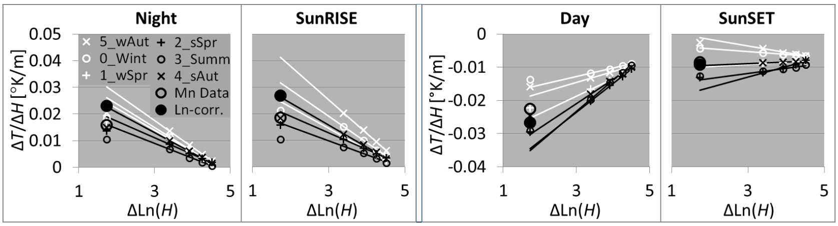


FIG. 4.1

The panels for each of the four daily parts show that the temperature gradients follow reasonably well a linear trend between 20 m and 100 m in the half logarithmic representation and how the values for the lowest layer 0–20 m (at ΔLn = 1·74) deviate from it. At each layer height the means of the six seasonal temperature gradients correspond to those as used for the four daily cardinal parts. Here only the means of the temperature gradients in the lowest layer are shown, the original ones with big black circles, and the mean of the extrapolated logarithmic value as black discs.

$[\mathrm{Ln}(H_{lower}+1{\cdot}5)+\mathrm{Ln}(H_{upper}+1{\cdot}5)]/2$, and how the temperature gradients in the lowest layer deviate from the logarithmic trend as shown with the linear regression lines. This effect was, however, found to be small. It *increases*, *e.g.*, the seasonal mean of the night-time temperature gradients by +7·3 K/km, and those at sunrise by +8·4 K/km; it *reduces* those at daytime by –4·2 K/km, and those at sunset by –0·8 K/km. Fig. 4.2 shows their effect on the resulting refractions. Because the differences are less than one arcmin and similar in size to other neglected contributions (see 6*ii*) this correction has not been applied to the individual T'_o profiles but to each one of the local hours (Fig. 2), of the six seasons (Fig. 3), and of the four cardinal diurnal parts of the day (Fig. 9). Note that the temperature gradients for the four cardinal parts of the day are first calculated separately for each of the six seasons and by selecting afterwards at each height (layer) the mean of the six seasons.

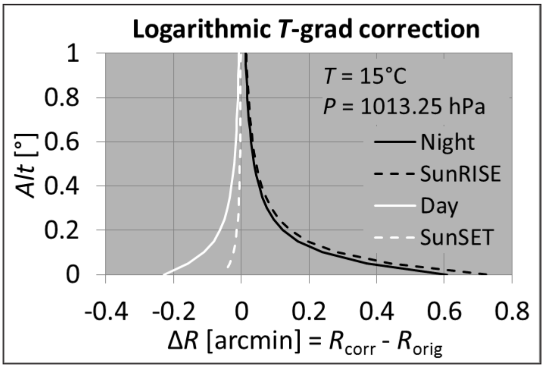


FIG. 4.2

Refraction differences resulting from replacing in the lowest layer the original temperature gradient, as derived from the sounding data, with the logarithmically-corrected one.

5. Refraction calculation

Having found that the temperature gradients resulting from the statistical analysis of the sounding data describe their general diurnal and seasonal pattern, this gives some confidence that this also applies to the refractions obtained from the ray-tracing calculations which use them. These calculations assume stratified atmospheric air layers. That may be questioned for the land/sea transition at coastal locations. However, because the temperature-gradient profiles represent mean values of different locations, this effect tends not to be a systematic one but is rather expected to increase the variabilities somewhat.

For ray-tracing calculations the author uses his program REF_CALC which is a multi-layer adaptation of the one described by Hohenkerk & Sinclair[17] with various improvements. Noteworthy is the index of refraction, which is calculated with the 4-parameter Peck–Reeder formula[38], and gravity, which is calculated according to Hinze *et al.*[39], as a sum of the following three contributions: the gravity at ellipsoidal latitude, g_T (their eq. 2), the atmospheric correction, δg_{atm} (their eq. 3), and the height correction, δg_h (their eq. 5). In order to avoid a discontinuity at 10 km where their quadratic approximation for δg_{atm} starts erroneously to increase, their function has been approximated in its valid range with an exponentially decreasing one. The layer's relevant gravity is found iteratively within the layer at its mid-pressure height.

A layer with constant temperature isolates water vapour to penetrate from the layer below to the layers above. In order to prevent calculating such cases in the lower atmosphere, the program replaces a zero temperature gradient with a very small value. For this study REF_CALC calculated the refractions at standard conditions using for the water-vapour pressure the same power-law equation as proposed by Hohenkerk & Sinclair[17]. Van der Werf shows[18] that the Clausius–Clapeyron equation agrees better over a wider temperature range with measured water-vapour pressures than the power-law equation. At standard conditions this difference is marginal, less than an arcsec. However, for determining the refraction conversion function for a large temperature range the 4-parameter Clausius–Clapeyron equation (eq. 28 in ref. 18) together with Ciddor's[40] refractivity index was used. The program has been thoroughly

TABLE III

Selected parameters for the Std and NA conditions as used in this study.

Standard (Std) Condition		Nautical Almanac (NA) Condition
@ Surface:	@ Observer:	All as at Std condition, except
g [ms^{-2}] = 9·806190	HoE [m] = 2	@ Observer:
r_c [m] = 6378092	P [hPa] = 1013·25	P [hPa] = 1010
	T [°C] = 15	T [°C] = 10
	$R.H.$ [%] = 75	n–1 = 2·8101645E-4
	λ [nm] = 570	
	n–1 = 2·7689572E-4	

benchmark tested with three other programs with maximum differences of a few arcsec for extreme conditions as possibly applicable in this study.

Most refraction calculations have been done for standard conditions, 'Std', and a reduced number also for the conditions, 'NA', often used in nautical almanacs, with both as defined in Table III. The selected parameters assume an observer at mid-latitude (± 45°) and viewing in mid-azimuth direction (± 45°).

6. *Temperature, pressure, and other parameters*

(*i*) Converting refractions to different atmospheric pressure and temperature. At altitudes above about 10 degrees the changes in density and consequently the resulting changes in refraction agree with the corresponding dependencies of refractivity of air from the same parameters, *i.e.*, they are to a large extent proportional to pressure (better than an arcsec) and inversely proportional to the absolute temperature (up to a few arcsec). At low altitudes it was found that this relationship is no longer sufficiently accurate. Bessel[41] used the exponents A (here κ) and λ (here –λ) which depend on the apparent altitude Alt:

$$R_x/R_{Std} = (P_x/P_{Std})^{\kappa} \cdot (T_x/T_{Std})^{\lambda}. \tag{1}$$

In Bessel's study[41] these exponents reach at $Alt = 0{\cdot}5°$ values of $\kappa = 1{\cdot}0780$ and of $\lambda = -1{\cdot}5789$. Van der Werf[18] shows in his Fig. 5c that for a large altitude range the dependency of the P and T conversion on altitude can be described with an altitude compression/decompression, his eq. 51:

$$\tan(Alt_x) = (P_x/P_{Std})^{\mu} \cdot (T_{Std}/T_x)^{\nu} \cdot \tan(Alt). \tag{2}$$

The two panels of Fig. 5 show how the powers κ and λ depend on altitude and how the powers κ and λ start below about $Alt \approx 0{\cdot}5°$ to depend additionally also on the diurnal temperature-gradient variations close to the surface. The powers are calculated with R_L and constant relative humidity, $R.H.$ In panel b one can also see why the astronomers, who observe at higher altitudes, use $\lambda = -1$ whereas the surveyors, who tend to observe near $Alt = 0°$, use $\lambda = -2$.

Here it is found that for the selected pressure, temperature, and altitude ranges the use of correction functions with polynomials would be easier to use than a solution with the powers κ and λ or with the tangent function. With u relating either to Std or NA condition, and k relating to one of the four diurnal temperature-gradient profiles (Tgrad) the following procedure has been selected for converting at a given HoE the LPAARs in the range $0° \leq Alt \leq 10°$:

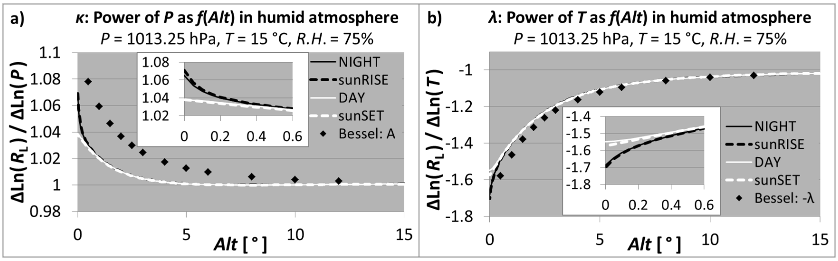


FIG. 5

Dependence of the apparent astronomical refraction, R, at low altitudes on pressure (panel a), and temperature (panel b). Below about $Alt = 0{\cdot}5°$ the diurnal influences become apparent. The refractions for the four cardinal parts of the day correspond to those described in section 7 ii. The black diamonds show for comparison a selection of Bessel's values; his A and λ (here κ and $-\lambda$); Bessel related his values to the conditions used in his refraction table, *i.e.*, $P = 29{\cdot}6$ inches of mercury, $T = 48{\cdot}75$°F (1002·4 hPa, 9·3°C), and neglecting *R.H.*

Temperatures:

With T_u and T_x in K and with apparent altitude, *Alt*, and refraction, R, in angular degrees, and with the 'normal' (inverse) ratio η_{Tu} being:

$$\eta_{Tu} = (T_u / T_x) \tag{3.1}$$

using additionally also a correction function f_{T_corr} evaluated for the relevant temperature gradient profile T_{grad_k}, then:

$$R(T_x) = R(T_u) \cdot \eta_{Tu} \cdot f_{T\_corr\_u\_Tgrad\_k}\,(Alt, \eta_{Tu}) \tag{3.2}$$

with

$$f_{T\_corr\_u\_Tgrad\_k}\,(Alt, \eta_{Tu}) = \Sigma\, A_i \cdot Alt^i \;\text{ with i} = 0 \text{ to } m_{Tu} \tag{3.3}$$

and with

$$A_i = \Sigma\, a_j \cdot \eta_{Tu}{}^j \;\text{ with j} = 0 \text{ to } n_{Tu}. \tag{3.4}$$

Pressures:

With P_u and P_x in hPa and *Alt* and R in angular degrees, and with the ratio η_{Pu} being:

$$\eta_{Pu} = (P_x / P_u) \tag{4.1}$$

using the correction function f_{P_corr} evaluated for a particular typical temperature gradient profile T_{grad_k}, then:

$$R(P_x) = R(P_u) \cdot \eta_{Pu} \cdot f_{P\_corr\_u\_Tgrad\_k}\,(Alt, \eta_{Pu}) \tag{4.2}$$

with

$$f_{P\_corr\_u\_Tgrad\_k}\,(Alt, \eta_{Pu}) = \Sigma\, A_i \cdot Alt^i \;\text{ with } i = 0 \text{ to } m_{Pu} \tag{4.3}$$

and with

$$A_i = \Sigma\, a_j \cdot \eta_{Pu}{}^j \;\text{ with j} = 0 \text{ to } n_{Pu}. \tag{4.4}$$

The polynomial degrees m and n have been selected on the following basis: the polynomial approximations have near $Alt = 0°$ the largest differences to the ray-tracing results; these differences have over all the selected parameter range (temperature from −50°C to +50°C and pressure from 870 hPa to 1084 hPa) a standard deviation designated as $\sigma_{Alt \simeq 0}$. The polynomial conversion function is intended to be used until an apparent altitude of 10 degrees. Above, at $Alt \geq 10°$, the conversions will be made by using only the parameter ratio η resulting in refraction differences (compared to the ray-tracing values) having a standard deviation designated as $\sigma_{Alt = 10}$. The degrees m and n are now selected so that $\sigma_{Alt \simeq 0} \approx \sigma_{Alt = 10}$ for each of the four daily cardinal T' profiles.

The polynomial parameters are determined with least-squares fits to ray-tracing results calculated in altitude intervals of 3 arcmin in the range 10° > Alt > 3° and of 0·12 arcmin in the range 3° > Alt > −0°·026, this for each of the cardinal T' profiles for 25 different temperatures and for 17 different pressures in their corresponding ranges. (Note that for the temperature a 'global' range from −50°C to +50°C was selected, realizing only after a helpful hint that the high temperature in combination with the standard $R.H.$ = 75% could probably not be survived.)

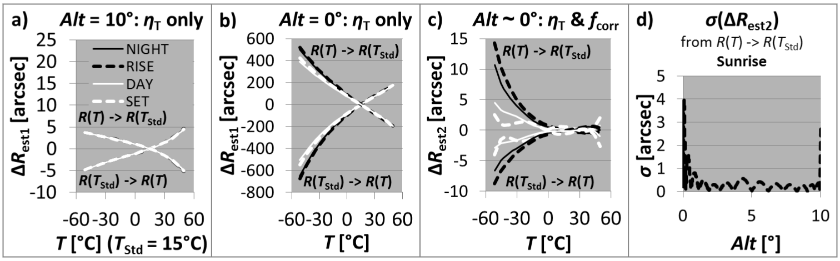


FIG. 6

The four panels show the effect which the correction function has when converting refractions between different temperatures. Panels *a* and *b* show the refraction differences between ray-tracing results and converted refractions when using only the temperature ratio, η. Panel *c* shows how the correction function reduces the large differences in panel *b*, and panel *d* shows for the sunrise conversions the standard deviations of all converted refractions in the altitude range at $0° \leq Alt \leq 10°$ over the temperature range $-50°C \leq T \leq +50°C$.

Fig. 6 shows the various refraction differences between the ray-tracing results and the results from converting refractions $R(T)$ from/to a temperature T to/from T_{Std} and the effect of the corrective factors $f_{corr\_u\_Tgrad\_k}$ on this conversion. Panel *a* shows the refraction differences $\Delta R_{est1} = R_{est1} - R_{calc}$ at $Alt = 10°$ when converting only with η_T. Panel *b* shows the large values of ΔR_{est1} when converting at $Alt = 0°$ only with η_T. Panel *c* shows at $Alt \simeq 0°$ the maximum refraction differences $\Delta R_{est2} = R_{est2} - R_{calc}$ when converting additionally to η_T also with f_{corr}. The differences in this panel *c* are similar to or better than the differences at $Alt = 10°$, except those at night and during sunrise which show somewhat larger differences below about −25°C. The results in this panel show how well, with the polynomial orders listed in Table IV, the set goal of $\sigma_{Alt \simeq 0} \approx \sigma_{Alt = 10}$ was reached. Panel *d* contains another representation on how well the set goal was reached using here only the sunrise results. The panel shows the standard deviation, σ, of all the 48 refraction differences ΔR_{est2} from the conversions $R(T)$ to $R(T_{Std})$ and $R(T_{Std})$ to $R(T)$. The variability of 2·7 arcsec

TABLE IV

Polynomial degrees for calculating the conversion functions $f_{corr\_u\_Tgrad\_k}$ with the maximum standard deviations, $\sigma_{Alt\approx 0}$, of the refraction differences $\Delta R(\eta) = R_{est2} - R_{calc}$ over all the η-range when converting to and from T_{Std} or P_{Std}.

T'-profile	Temperature, $\sigma_{Alt=10}\simeq 2{\cdot}7''$			Pressure, $\sigma_{Alt=10}\simeq 0{\cdot}12''$		
k:	$m_{Tk}(Alt)$	$n_{Tk}(\eta_{Tk})$	$\sigma_{Alt\approx 0}['']$	$m_{Pk}(Alt)$	$n_{Pk}(\eta_{Pk})$	$\sigma_{Alt\approx 0}['']$
NIGHT-time	13	5	3·1	13	4	0·33
SunRISE	13	5	4·0	13	4	0·42
DAYtime	7	5	1·4	7	4	0·12
SunSET	4	4	1·2	4	2	0·05

at $Alt = 10°$ is the one when converting only with η_T. The smaller variability below 10° altitude shows the effect from $f_{corr\_u\_Tgrad\_k}$; the variabilities oscillate at small values and start to increase when approaching $Alt = 0°$ reaching a maximum of 4 arcsec at 0·074°.

(*ii*) Neglected parameters.

Differences in relative humidity, *R.H.*, wavelength, λ, or gravity, *g*, are not converted because they were either unknown or neglected. At standard conditions and at $Alt = 0°$ the refraction difference between *R.H.* 0% and 100% is about +15 and –5 arcsec, respectively, relative to the selected 75%. Refractions at wavelengths in the range between 485 nm and 713 nm differ at $Alt = 0°$ no more than about ±17 arcsec from the selected 570 nm. The world-wide minimum surface gravity appears to be 9·7639 ms^{-2} and the maximum one 9·8337 ms^{-2}; these two extreme values result in a refraction difference at $Alt = 0°$ of –8 and +6 arcsec, respectively, compared to the selected 9·8062 ms^{-2}. These various neglected contributions indicate that without considering these parameters it becomes difficult to achieve overall accuracies which are much better than about a minute of arc. They compare at $Alt = 0°$ — as found in this study — with the maximum range of diurnal refraction differences of about 6 arcmin, and of about 1·3 arcmin for the maximum seasonal range.

Using above 20 km in all atmospheric models the temperature gradients from the standard atmosphere[42] neglects its contribution to the seasonal variability. The size of this neglected contribution has been estimated by calculating the refractions with 12 monthly temperature-gradient profiles which contain below 20 km elevation the (now constant) one from the standard atmosphere and above, between 20 km and 120 km, the temperature gradients resulting from the monthly zonal mean temperatures for latitude 45°N of the COSPAR International Reference Atmosphere (CIRA-86 with last modified data from 2014)[43]. Using these temperature gradients the resulting monthly refractions vary at $Alt = 0°$ only by $\sigma = \pm 0{\cdot}2$ arcsec.

(*iii*) Height of Eye (*HoE*).

This study uses a standard height of eye of 2 m above the Earth's surface. The effect of a different *HoE* on the astronomical refraction at standard condition was determined for a *HoE* of 10 m ('Low') and of 50 m ('High'). For this comparison it is assumed that the observer reads the somewhat smaller air pressure but the same temperature and the same humidity as at 2 m (15°C and 75%). Fig. 7 shows that some of the diurnal differences are sufficiently large and have thus to be taken into account in the subsequent reanalyses of

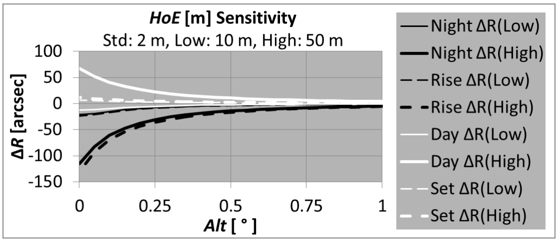


FIG. 7

Refraction differences, ΔR, for observers at $HoE = 10$ m (Low, thin lines) and 50 m (High, thick lines) compared to those observed at the standard height of 2 m. All differences from $HoE = 10$ m are roughly in the range of other neglected contributions but not those from $HoE = 50$ m, except the one for sunset where the temperature-gradient profile is almost constant near the dry adiabatic lapse rate.

the Königsberg observations made at an estimated *HoE* of about 50 m. The differences come mainly from the observer looking through different parts of the temperature-gradient profile with maximum differences where the profile is most non-linear.

7. Refractions from typical temperature-gradient profiles

(*i*) Diurnal refraction variation — LPAARs as a function of *LHr*.

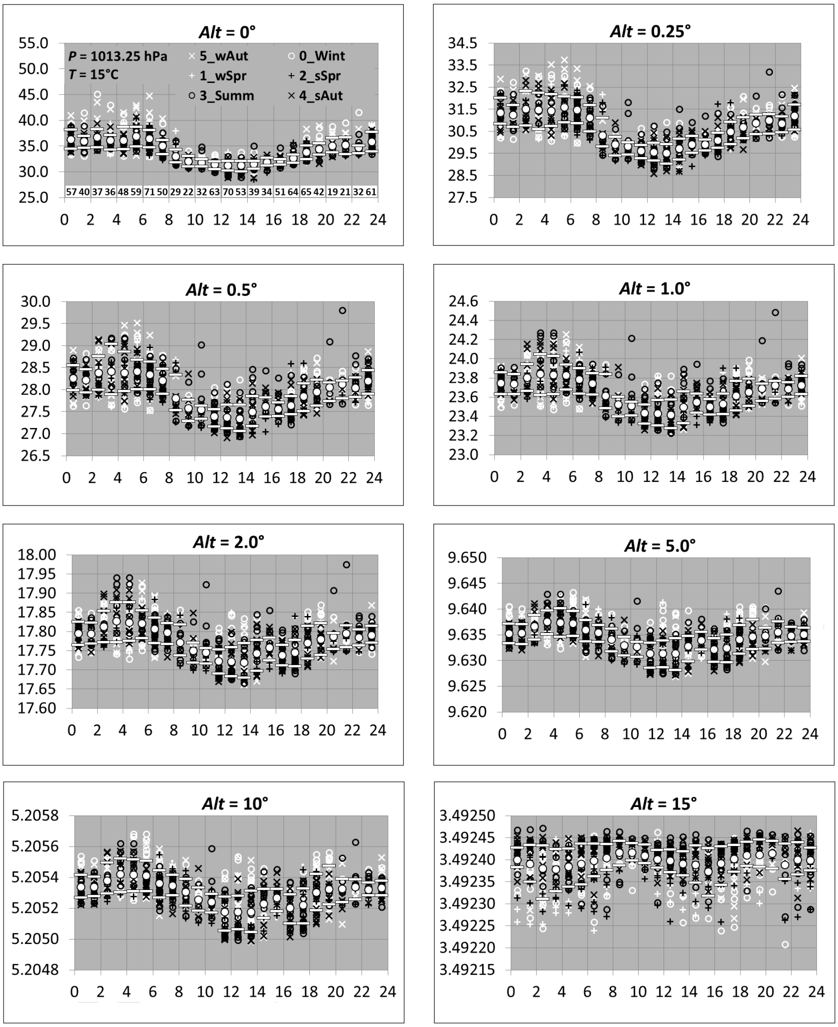
For visualizing the refraction's diurnal cycle, all $R_o(T'_o)$ are shown in Fig. 8 as a function of their local hours together with the mean, R_h, and its variability limits at each local hour. The first panel with the values at $Alt = 0°$ shows that during night-time the refractions have maximum mean values around 36 arcmin and during daytime about 5 arcmin less. The refractions react about one local hour delayed on the night-to-day transition and are about 3 local hours after sunrise already close to the daily minimum, which is about ½ to 2 local hours after Sun transit. From this minimum, the refractions increase gradually until reaching again the maximum level about ½ hour before local midnight. At other altitudes the refractions show similar behaviour, except that they tend to follow a more sinusoidal shape. At the higher altitudes, 15° and 30°, the diurnal variations vanish in the 'noise'.

Depending on how the light-ray crosses the temperature-gradient profile, *i.e.*, depending on its angle of incidence, the outliers are differently distributed; at low altitudes outliers are prone to higher refractions whereas at high altitudes to lower ones.

From the results shown in this figure, a night-time and a daytime dataset have been defined which cover refractions within diurnal time spans where their variations appear to be reasonably small; for the night-time refractions this time span is $23 \leq LHr \leq 7{+}24$ and for the daytime ones $9 \leq LHr \leq 17$.

(*ii*) Temperature-gradient profiles and refractions at the four cardinal parts of the day.

The T' profile for each of the four daily cardinal parts results from calculating at each layer first the mean from all soundings for each of the six seasons, *i.e.*, by determining first the mean gradient of, *e.g.*, all night-time gradients from all soundings during season 5_wAut, *etc.*, but replacing the temperature gradient



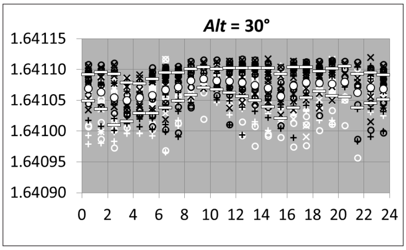


FIG. 8

Each panel shows for a selected altitude all individual refractions from the sounding data (with their seasonal symbol) in arcmin as a function of the local hours, and for each local hour also the mean of all these individual refractions (white discs) together with their ⅙ and ⅚ quantile limits (white horizontal bars). The small numbers in the first panel above the local hours indicate the number of data. The results reflect (*a*) the increased refraction and their increased variability during the night-time inversions, (*b*) that the refractions are further prone to outliers during the winter season (white data points), (*c*) the reduced refractions and their noticeably reduced variability under solar heating during the day, and (*d*) the reduced variability at increased altitudes.

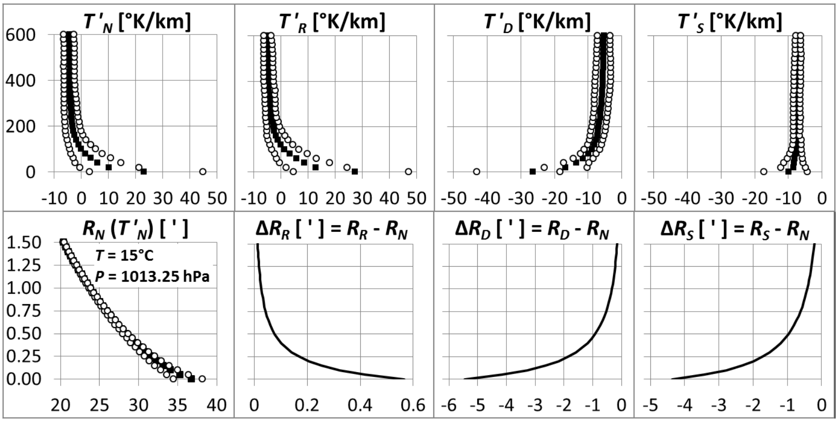


FIG. 9

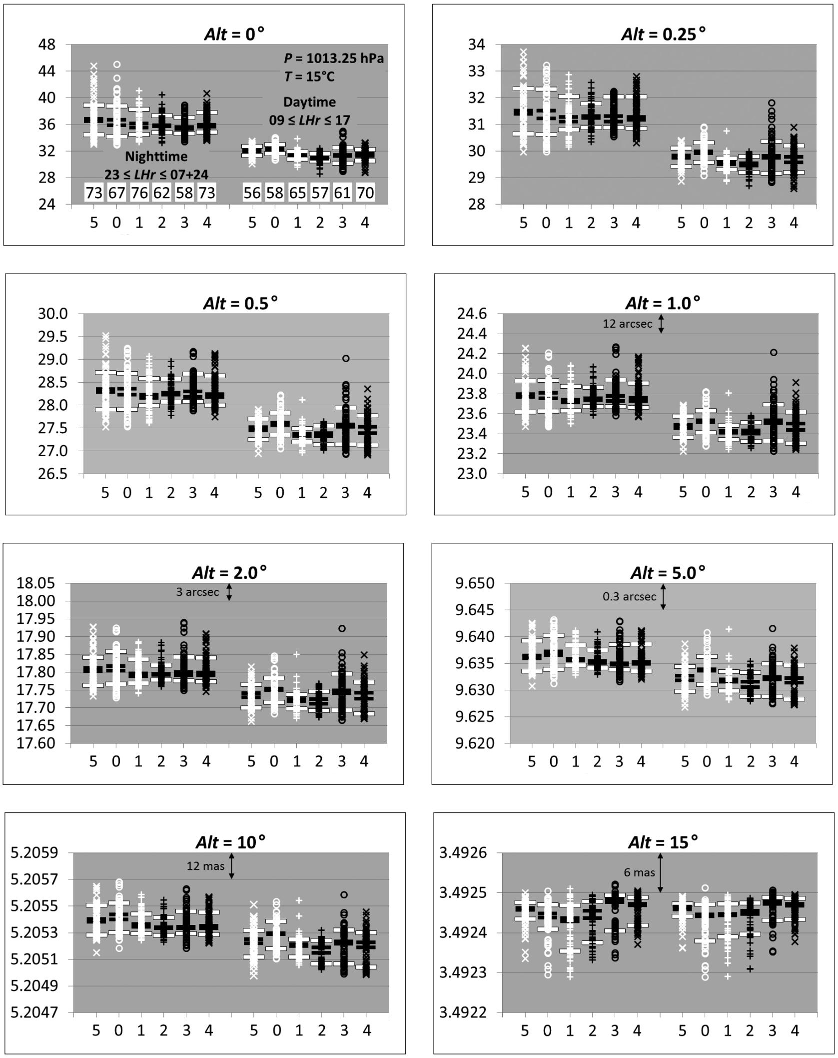
The four panels in the upper row show with the black squares the temperature gradients [K/km] below 600 m above station derived for (left to right) the night-time, sunrise, the daytime, and sunset; the circles correspond to the two quantile limits ⅙ and ⅚. The temperature gradient in the lowest layer is the one from the logarithmic correction. Note that the temperature-gradient profile at sunset agrees best with a constant one as used in models of the standard atmosphere. The first panel in the lower row shows with the black squares the mean night-time refractions in arcmin calculated for altitudes below 1·5° with its T' profile in the panel above; the circles indicate again the ⅙ and ⅚ quantile limits. The three panels to the right show how in the same altitude range the sunrise refractions, daytime refractions, and sunset refractions differ in arcmin from the night-time ones.

in the lowest layer with the logarithmically-corrected one (as described earlier), and by taking afterwards the mean of all six seasonal night-time temperature gradients. With these T' profiles the refractions have been calculated at standard conditions; the refraction variability is estimated with the ⅙ and ⅚ quantile limits from all the individual refractions $R_0(T'_0)$ contributing to the corresponding local-hour range. Fig. 9 shows that at $Alt = 0°$ the maximum diurnal refraction differences are 6·0 arcmin between sunrise and daytime and 4·9 arcmin between sunrise and sunset.

(*iii*) Seasonal and other LPAAR variabilities resulting from the radiosonde data.

The large number of data invites one to perform gross estimates of some of the contributions which add to the total refraction variability. This is done with the large daytime and night-time datasets. One has, however, to be aware that the refraction variabilities derived from the radiosonde data are expected to be smaller than those from individual day-to-day observations because the former are calculated with T' profiles which are themselves already a mean of at least ten soundings. The results may, however, give an indication of their relative size. Comparing these variabilities with those from directly measured LPAARs (see 9*iv*) gives an indication by how much those from the soundings have to be increased to make them comparable with those of individual observations.

The most obvious variability is the one of the individual refraction results which is mainly caused by different meteorological conditions, *i.e.*, meteorological differences *beyond* different temperatures and pressures at the



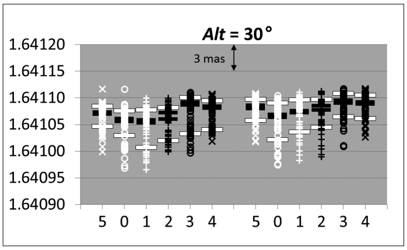


FIG. 10.1

The panels show at selected altitudes the refractions from the sounding data in arcmin for each season (see codes in Table II); the left part of each panel contains the night-time refractions and the right part the daytime ones. The mean refractions are not shown but their ⅙ and ⅚ quantiles as white bars. The black bars show the two quantile limits (⅙ and ⅚) of the 14 refraction results obtained from jack-knifing the temperature gradients in the original data set. The small numbers shown in the first panel above the *x*-axis indicate the total number of data. At the lowest altitudes the night-time refractions are considerably larger than the daytime ones. With increasing altitude the night/day difference diminishes and disappears in the noise at higher altitudes.

observer which are here kept constant at standard conditions. These large variations are designated here as 'meteorological' and their quantile limits ⅙ and ⅚ are shown in Fig. 10.1 for each season with the white bars. The individual data points in this figure correspond to the refraction $R_0(T'_0)$ which contributes during a season to one of the local hours within the night-time or daytime ranges. At $Alt = 0°$ the 1·5 MAD variability of all 409 night-time refractions is ±2·1 arcmin (5·7%) compared to ±0·86 arcmin (2·7%) of all 367 daytime refractions; at night the refraction's variability is thus almost 2·5 times larger than during the day. These values contain a small variability contribution of ±0·38 (night) and ±0·34 arcmin (day) corresponding to the standard deviation of all mean values shown in Fig. 8 at the different local hours which make up the corresponding night-time and daytime ranges; these latter contributions are designated as 'hourly'. The 'meteorological' variabilities are expected to contain also variability contributions from seasonal and local differences which are estimated separately as follows: It is hypothesised that by averaging a set of 'jack-knifed'[44] refraction results from a certain season would remove the individual meteorological properties of the data in the set but would keep some general ones which are expected to be more representative for the corresponding season than analysing the variability 'directly' from, *e.g.*, the median (middle values of the white bars). For estimating the 'seasonal' contribution all temperature gradients for the night-time and daytime part of each season are therefore jack-knifed by attributing randomly to each temperature gradient profile T'_0 within the full-sized dataset a '0' or a '1' resulting in two jack-knifed datasets '0' and '1'; six such different splits thus produce 12 jack-knifed datasets plus the original full-sized dataset which is counted twice, *i.e.*, j = 12+2 temperature-gradient profiles which result in a mean temperature-gradient profile T'_j and their refractions R_j for the night-time and daytime of each season. In Fig. 10.1 the variability from the jack-knifed results is shown with black bars corresponding to their two quantile limits ⅙ and ⅚ of the 12+2 values. It finally turned out that these jack-knifed variabilities are only marginally larger than the 'directly' obtained ones shown in parenthesis. The median of the six seasons, *i.e.*, the middle of the two black bars, and their 1·5 MAD variability are at $Alt = 0°$ for the night-time 35·77 ±0·31(36·30 ±0·31) arcmin and for the daytime 31·36 ±0·35 (31·31 ±0·24) arcmin, and the maximum ranges, *i.e.*, the difference between the two seasons with the maximum and the minimum refractions are 1·2 (0·8) and 1·3 (1·2) arcmin for night-time and daytime, respectively. Note that these 'seasonal' variability contributions do not differ much from the 'hourly' ones, thus suggesting that the effective seasonal contributions may even be less. On the other hand the daytime refractions may contain some contribution caused by the seasonally different lengths of the local hours.

Only the daytime seasonal refractions indicate having a systematic pattern in the altitude range $0° \le Alt \le 10°$ consisting of a maximum refraction during High Winter, '0_Wint', which applies to all 106 (106) calculated altitudes in the range, and a minimal refraction during Late Spring, '2_sSpr', which applies to 97 (98) of the 106 altitudes; at 9 (8) altitudes in the range $0{\cdot}5° < Alt < 1°$ Early Spring values, '1_wSpr', are marginally smaller. The night-time refractions give at some of the altitudes a vague indication of a seasonal dependency but they do not appear to have common systematics. At $Alt = 0°$ their maximum range of 1·2 (0·8) arcmin is between maximum in Late Autumn '5_wAut', and minimal in High Summer, '3_Summ'.

The temperature-gradient selection in the jack-knifed datasets (Fig. 10.1) is partially also a selection of different locations; the variability of the jack-knifed results is therefore expected to contain at least partially also a 'local' variability contribution. Its possible maximum size is estimated as the 1·5 MAD variability of the six seasonal half-quantile-ranges from the jack-knifed results (black bars). At $Alt = 0°$ one obtains for the night-time refractions ±0·24 arcmin and for the daytime ones about ±0·10 arcmin, thus again at night about 2·5 times larger than during the day, as already observed for the 'meteorological' variability.

The various variability contributions are shown in Fig. 10.2 as a function of altitude. Note that all the various estimated small variabilities can only be indicative because several small refraction contributions from neglected parameters would have similar or even somewhat larger contributions.

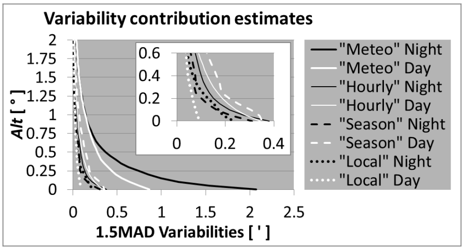


FIG. 10.2

Gross variability estimates for the 'meteorological', 'hourly', 'seasonal', and 'local' contributions at $Alt \leq 2°$ as described in the text. For individual observations these variabilities are expected to be larger (see 9 *iv*).

8. Observed LPAARs

The following LPAAR observations have been considered for comparing in the next main section with those derived from the sounding data:

(*i*) Star and Sun observations at Bessel's observatory in Königsberg.

At the observatory in Königsberg (today Kaliningrad) Argelander performed in 1820/21 a series of star and solar observations for obtaining better refraction values at zenith distances between 85° and 90°. He made those observations with the observatory's *Cary* altazimuth circle. His observations for both years are published in ref. 8 (pages IX to XII); the air temperatures in °F for those observations follow on pages XII – XIII and his results for the star and the sunset observations on page XIV. Sidereal times and air pressures were found with the documentation of the observations performed on the same day with the observatory's meridian telescope and which have been published for 1820[45] and for 1821[46]. For the refraction observations the mean of all recorded air pressures on the same day was used. For a small number of observations no atmospheric conditions were available; they were then estimated from the nearest ones. The available data appeared to be sufficient for performing a reanalysis.

Knowing the location, the day, the star, its true altitude, $Alt_{\rm true} = Alt_{\rm app} - R_{\rm obs}$, and the atmospheric conditions allows estimation of the local time of the observation; this, however, only if the ambiguity of the two possible results for rise and set can be solved. Applying several criteria indicated that all observations appear to have been done while the observed object was setting. A confirmation of this was later found in the introduction to the *Tabulae Regiomontanae*[41] where Bessel mentions that Argelander observed the stars when nearly setting. As a consequence of that it was found that Argelander performed his LPAAR observations during sunset, dusk, and night.

In Table V Argelander's results are compared with those from a reanalysis of his published observed refractions, R_{obs}, when applying this study's different P/T conversions for the four cardinal parts of the day and linearly interpolating between their applicable *LHr* ranges. Note that Argelander did not directly mention the mean refractions from his measurements, but rather how the tabular values for the refractions in the *Fundamenta*, R_{Fund}, differed by ΔR_{Fund} from his mean refractions R_{Arg} and their *probable* error ¦Δ¦ resulting from his N_{Arg} observations. His refractions related therefore to the atmospheric standard conditions as used in Bessel's *Fundamenta*[7], *i.e.*, $T = 48{\cdot}75$°F and $P = 29{\cdot}6$ inches of mercury ($T = 9{\cdot}3$°C and $P = 1002{\cdot}4$ hPa); the values listed in Table V under this heading relate to those published by Bessel in his *Fundamenta* and by Argelander as a function of ZD. For comparing his results with those from the reanalysis (N_{Re} and R_{Re}) they have been converted to NA condition. It appears that there are some small differences in the number of star observations. This would suggest that Argelander removed some of them in his analysis. There is one case where the reanalysis has one less but I do not remember

TABLE V

Comparison of the low-altitude refractions from Argelander's star and sunset observations, as published in ref. 8, page XIV, with those from this reanalysis.

Star observations:

$T = 9{\cdot}3$°C, $P = 1002{\cdot}4$ hPa						$T = 10$°C, $P = 1010$ hPa						
ZD [°]	$R_{Fund.}$ [']	$\Delta R_{Fund.}$ ["]	N_{Arg}	$R_{Arg.}$ [']	±¦Δ¦ ["]	*Alt* [°]	$R_{Arg.}$ [']	±¦Δ¦ ["]	N_{Re}	R_{Re} [']	±¦Δ¦ ["]	ΔR ["]
85·0	9·74	−5·96	47	9·84	4·89	5·0	**9·89**	4·9	47	**9·83**	4·0	**3·8**
85·5	10·60	−9·21	50	10·76	4·76	4·5	**10·81**	4·8	50	**10·74**	4·2	**4·2**
86·0	11·62	−7·88	56	11·75	4·85	4·0	**11·80**	4·9	56	**11·73**	5·3	**4·5**
86·5	12·82	−5·14	63	12·90	5·10	3·5	**12·97**	5·1	64	**12·91**	4·5	**3·1**
87·0	14·27	−4·49	75	14·34	5·84	3·0	**14·41**	5·9	76	**14·33**	4·7	**4·6**
87·5	16·04	−4·72	75	16·11	6·87	2·5	**16·19**	6·9	75	**16·10**	6·8	**5·4**
88·0	18·23	−0·45	82	18·24	8·89	2·0	**18·32**	8·9	81	**18·23**	7·5	**5·8**
88·5	21·02	4·44	80	20·95	11·45	1·5	**21·04**	11·5	82	**20·94**	9·3	**5·8**
89·0	24·64	7·69	51	24·51	17·40	1·0	**24·61**	17·5	55	**24·57**	17·2	**2·9**
89·5	29·46	18·15	30	29·16	20·48	0·5	**29·28**	20·6	34	**29·20**	21·8	**4·8**

Sun observations:

$T = 9{\cdot}3$°C, $P = 1002{\cdot}4$ hPa						$T = 10$°C, $P = 1010$ hPa						
ZD [°]	$R_{Fund.}$ [']	$\Delta R_{Fund.}$ ["]	N_{Arg}	$R_{Arg.}$ [']	±¦Δ¦ ["]	*Alt* [°]	$R_{Arg.}$ [']	±¦Δ¦ ["]	N_{Re}	R_{Re} [']	±¦Δ¦ ["]	ΔR ["]
85·0	9·74	−4·96	7	9·83	2·52	5·0	**9·87**	2·5	7	**9·79**	2·2	**5·2**
85·5	10·60	−3·98	9	10·67	1·74	4·5	**10·72**	1·7	9	**10·63**	0·6	**5·5**
86·0	11·62	−4·60	10	11·69	2·31	4·0	**11·75**	2·3	10	**11·67**	3·4	**4·6**
86·5	12·82	−5·18	12	12·90	3·85	3·5	**12·96**	3·9	12	**12·87**	5·4	**5·7**
87·0	14·27	−2·71	13	14·31	5·31	3·0	**14·38**	5·3	13	**14·27**	5·5	**6·1**
87·5	16·04	0·29	13	16·03	4·91	2·5	**16·10**	4·9	13	**16·00**	5·9	**6·2**
88·0	18·23	5·72	12	18·14	5·20	2·0	**18·22**	5·2	12	**18·11**	4·8	**6·2**
88·5	21·02	14·13	12	20·79	7·32	1·5	**20·87**	7·4	12	**20·76**	8·4	**6·6**
89·0	24·64	30·60	12	24·13	15·41	1·0	**24·22**	15·5	12	**24·05**	13·8	**10·3**
89·5	29·46	48·60	9	28·65	31·60	0·5	**28·76**	31·7	9	**28·51**	26·6	**14·7**

having removed any of his observations. The maximum difference between the refractions of Argelander's *star* results and those from the reanalysis are 6 arcsec, and the probable errors of them at $Alt = 0{\cdot}5°$ are comparable, 21 and 22 arcsec for Argelander's results and those of the reanalysis, respectively. The numbers of *sunset* observations do all agree. The maximum difference between the two refraction results is 15 arcsec, and the probable errors of them are at $Alt = 0{\cdot}5°$ according to Argelander 32 arcsec and for the reanalysis 27 arcsec. The slightly increased differences for the sunset observations could possibly result from the smaller number of solar observations or from using a less accurate value for the Sun's parallax when calculating its true altitude. Note that the reanalysis contains additionally nine star observations at $Alt = 0{\cdot}25°$, and five sunset observations, three at $Alt = 0{\cdot}25°$ and two at $Alt = 0{\cdot}0°$; Bessel did not publish those observations; here they are shown in Table VIII.

(*ii*) Sunsets at Istanbul

The author performed between 2007 and 2011 photographic sunset observations over the horizon of the Marmara Sea (Köppen Csa). *HoE* varied between about 2·5 and 4·5 m with a few exceptions at about 20 m. It was assumed that the meteorological conditions at the observation sites in the area of Kartal/Maltepe corresponded to those as measured at the same coast about 25 to 30 km distant at the Atatürk Airport. For all observations the terrestrial refraction of the dip has first been adjusted to the atmospheric conditions of the observation and the resulting astronomical refraction has then been converted to Std and/or NA conditions. The dips and their terrestrial refraction were estimated with eq. 21*d* in ref. 2 using sea-surface temperatures (SST) corresponding to analysed gap-free SST data from OSTIA[47] which are derived from satellite data provided by the GHRSST project (https://www.ghrsst.org). Fig. 11 shows the refractions converted to NA conditions as a function of altitude, and the seasonal refractions of these observations are shown in 9*iii*. Note that the variabilities contain also those from the dip itself which contributes[1] under adiabatic conditions with an expected 1·5MAD variability of ± 0·3 arcmin and in inversions of ± 0·8 arcmin (see in ref. 1, Table 2, bold values for eqs. 11a and 11b).

When determining the astronomical refraction from observations relative to an apparent horizon, the observer obtains the apparent altitude, *Alt*, from measuring the angular distance, *HS*, between the Sun's limb and the apparent sea (or landscape) horizon and by deducting the angular distance between apparent and astronomical horizon, *i.e.*, its dip *D*:

$$Alt = HS - D. \qquad (5.1)$$

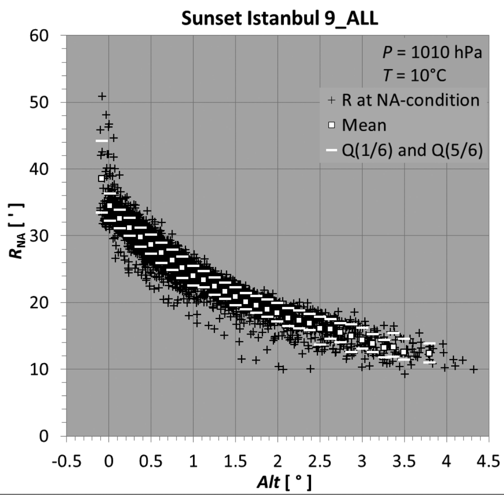
The apparent dip requires estimating the terrestrial refraction, R_T, between observer and horizon for obtaining its difference to the corresponding geometrical dip, D_g, as defined by the observer's *HoE* (see, *e.g.*, Fig. 1 in ref. 1):

$$D = D_g - R_T. \qquad (5.2)$$

Knowing the location and the time of observation allows calculation of the Sun's limb true (unrefracted) altitude, Alt_t, and with its difference to the apparent altitude, *Alt*, the astronomical refraction:

$$R = Alt - Alt_t. \qquad (5.3)$$

In case of atmospheric inversions the terrestrial refraction may reach abnormally high values, until infinite. An erroneous estimate of the terrestrial refraction may



Alt [°]	R_{NA} [']	*N*	$Q_{1/6}$ [']	$Q_{5/6}$ [']
-0.08	38.57	10	33.43	44.17
0.01	**34.50**	**127**	**32.23**	**36.31**
0.13	32.57	207	31.04	33.83
0.25	**31.20**	**362**	**29.93**	**32.68**
0.38	29.76	321	28.29	31.18
0.50	**28.62**	**283**	**27.28**	**30.15**
0.62	27.42	221	25.87	28.94
0.75	**26.28**	**194**	**24.81**	**27.88**
0.87	25.20	161	23.64	26.84
1.00	**23.97**	**148**	**22.53**	**25.47**
1.12	23.39	119	22.28	24.66
1.25	**22.38**	**118**	**21.09**	**23.71**
1.38	21.65	97	20.49	22.91
1.50	**20.86**	**102**	**19.52**	**22.24**
1.62	20.04	86	18.95	21.34
1.75	**19.56**	**84**	**18.40**	**20.97**
1.87	18.97	76	17.81	20.38
2.00	**18.40**	**74**	**17.16**	**19.88**
2.12	17.66	69	16.47	19.08
2.25	**17.26**	**62**	**16.07**	**18.77**
2.38	16.88	46	15.79	18.23
2.50	**16.05**	**47**	**13.67**	**17.79**
2.62	16.00	45	14.55	17.49
2.74	**15.44**	**33**	**14.04**	**16.90**
2.87	14.95	30	12.56	16.73
3.00	**14.37**	**25**	**13.09**	**16.21**
3.12	13.86	27	12.54	15.22
3.26	**13.21**	**17**	**11.77**	**14.37**
3.37	13.29	14	11.88	15.35
3.49	**12.55**	**11**	**11.40**	**14.51**
3.80	12.40	27	11.00	13.80

FIG. 11

Sunset refractions at Istanbul as a function of altitude. The mean altitude and mean refraction together with the two quantile limits ⅙ and ⅚ have been determined in intervals of 0·125° altitude and above 2·75° altitude in larger ranges; the figure shows, however, only the bold values and those at both ends.

thus have a strong impact on the resulting apparent altitude and consequently on the observed astronomical refraction.

(*iii*) Sampson's sunrise/sunset refraction observations at Edmonton, Alberta. Sampson documented his sunrise/sunset observations at Edmonton[12,13]; here only ref.12 is used because they allow estimation of the average atmospheric conditions. Edmonton has a (cold) continental Köppen class Dfc climate without large bodies of water in the neighbourhood. The landscape horizon varied in the range $-0{\cdot}09° \leq Alt \leq +0{\cdot}21°$. It is understood that his published refractions relate to observed temperatures and pressures adjusted to $Alt = 0°$. In his Table 3.3.1 he lists the mean refractions and the standard deviations of his corrected astronomical-refraction results. The average conditions for his observations are estimated from his Figures 3.5.5, 3.5.13, and 3.5.14. Table VI shows his refractions converted to the conditions used in this study.

TABLE VI

Sampson's observed refractions at Edmonton

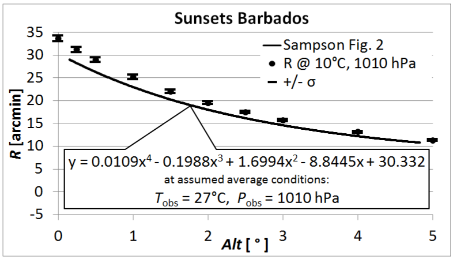
R @ $Alt = 0°$ [']	N	$P \simeq 935$ hPa	R_{obs}	$\simeq R_{NA}$	$\simeq R_{Std}$
SunRISE	234	$T \simeq -5°$C	42·8 ±11·0	42 ±11	41 ±11
SunSET	115	$T \simeq +7°$C	34·7 ±6·5	37 ±7	36 ±7

At this continental Köppen D climate location the refractions are greater than those found here for the coastal Köppen C locations. This is probably a result of the larger diurnal temperature ranges at the surface of a continental climate compared to a coastal one, resulting in greater temperature gradients and consequently also in somewhat greater refractions. Lapworth[37] found for

Cardington, which is like Edmonton also far from large bodies of water, that the constant temperature gradient occurs about 2 hours *before* sunset, which suggests that at sunset already small inversions may have developed at the surface which lead to greater refractions during sunset. The refraction difference between sunrise and sunset of about 5 arcmin agrees, however, with the one found here for the different coastal temperate climate (see 9*ii*). The seasonal variations of his LPAARs are discussed in 9*iii*.

(*iv*) Sampson's sunset refraction observations at Barbados, West Indies.

Sampson *et al.*[14] performed, from a coastal location on Barbados, photographic observations of the setting Sun from a *HoE* between about 2–3 m. Unfortunately they do not mention the meteorological conditions of their observations. However, since Barbados has a wet tropical climate (Köppen: Aw) the temperatures are expected to be fairly constant and their diurnal variations small. Assuming for the temperature at sunset a value of 27°C and a pressure agreeing with either NA or standard conditions may therefore be a reasonable estimate of the average conditions. The result from this approximate reanalysis is documented in Fig. 12. For comparing their observed refractions with those in this study the middle of their narrow point cloud shown in their Fig. 2 have been digitized (not the individual points), and then approximated by fitting through those values a 4th-order polynomial (continuous black line). Refractions at selected altitudes were then estimated with this polynomial and converted to NA conditions (black discs). The expected absolute variabilities (black bars) are calculated for these refractions using their relative refraction variability (their eq. 4).



Alt [°]	R_{obs} [']	R_{NA} [']	± σ [']
0	30.33	33.04	0.64
0.25	28.22	30.62	0.59
0.5	26.31	28.44	0.54
1	23.00	24.71	0.45
1.5	20.27	21.68	0.39
2	18.02	19.20	0.33
2.5	16.16	17.17	0.29
3	14.61	15.49	0.25
4	12.21	12.92	0.20
5	10.56	11.15	0.16

FIG. 12

This figure shows the result from the approximate reanalysis of observed sunset refractions on Barbados by Sampson *et al.*, as explained in the text. The values at *Alt* = 0° and 5° are slightly extrapolated.

(*v*) Hellerich's measurements of the Sun's vertical contraction.

Hellerich[11] aimed at obtaining LPAARs without the disturbing terrestrial refractions of the dip by measuring, in 1919/20 at the Seewarte in Kiel, the Sun's vertical contraction at large zenith distances. He mentions that the observations were done with a Dollond reflecting circle which had been mounted on a fixed stand, and that the instrument's nonius allowed a reading accuracy of 20 arcsec. The observations were done *without* a solar filter through a 15× magnifying telescope. He found that the observations of the horizontal diameter had a systematic bias of +0·23 arcmin which he took into account; the resulting vertical contractions have been listed together with the Sun's true (unrefracted) zenith distance at the time of observation; the publication comprises 212 sunset and 29 sunrise observations in the lowest 5 and 3 degrees altitude, respectively.

This study estimates from his measured differential refractions the apparent refractions, R, as a function of apparent altitude, Alt, by using a Bennett's-type[48] refraction formula, thus by finding with a weighted least-squares fit which uses the absolute residual as weight the two parameters A and B of the empirical function

$$R = \cot(Alt + A/(Alt + B)) \tag{6.1}$$

which Bennett found capable of well describing Garfinkel's refractions. The two panels of Fig. 13.1 show how the Sun's observed diameter contractions Δr_s compare with the corresponding fitted values at observed pressure and temperature and their absolute residuals $|\Delta R_{NA}|$. For the two datasets the fits result at NA conditions in the following parameters for the refraction approximations:

$$A_{rise} = 6{\cdot}526, \quad B_{rise} = 4{\cdot}155 \tag{6.2a}$$

$$A_{set} = 10{\cdot}523, \quad B_{set} = 6{\cdot}461 \tag{6.2b}$$

The refractions thus derived from his observations as a function of altitude are shown in Fig. 13.2.

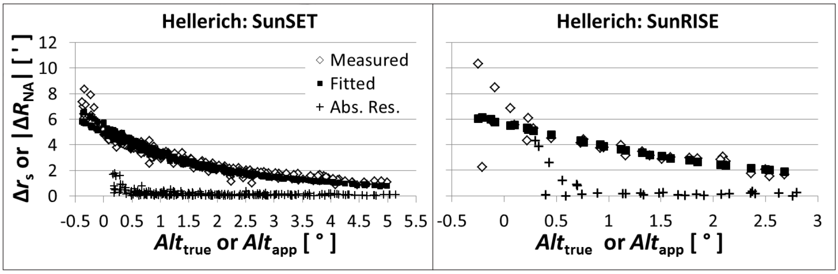
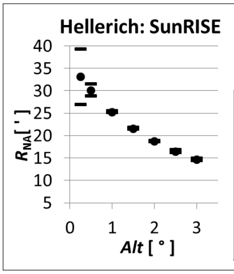


FIG. 13.1

The two panels show Hellerich's measured vertical Sun diameter contractions together with the corresponding estimate from the fitted cot-function. These data points relate to the true altitude. The panels show also the refractions absolute residuals relative to the fitted function; these data points relate to the apparent altitude.



Hellerich: SunRISE

Alt [°]	R_{NA} [']	1.5MAD [']
0.25	33.08	6.15
0.5	30.11	1.32
1	25.27	0.23
1.5	21.57	0.20
2	18.70	0.18
2.5	16.44	0.31
3	14.62	0.22

Hellerich: SunSET

Alt [°]	R_{NA} [']	1.5MAD [']
0.25	31.50	0.97
0.5	28.47	0.44
1	23.75	0.19
1.5	20.29	0.17
2	17.64	0.12
2.5	15.57	0.08
3	13.91	0.15
4	11.42	0.13
5	9.65	0.10

FIG. 13.2

Refractions at selected altitudes as they result from the reanalysis of Hellerich's observed compression of the Sun's vertical diameter.

(*vi*) Smiley's measurements of the Sun's vertical contraction.

Smiley measured refraction with the Sun's vertical diameter contraction near the horizon as observed with sextants during travels by ship through the Tropics[49], through Temperate Zones[50], and through Polar Regions[51]; some of them were also made in those climates from land. Note that at sea the observer's location could not be better known than as determined with sextant observations. He claimed that the measurements were made of the rising and setting Sun but appears not to have analysed their refractions separately. His publications contain no original measurements; a reanalysis was therefore not possible. Only the first publication with the refractions from the Tropics mentions also the refraction's *probable* error. All three publications show the results described as differences to some unknown refraction tables for 80°F, 50°F, 30°F, and 0°F. Only in the first two publications (Tropics and Temperate Zones) the resulting refractions have also been compiled in a table, both with the results from the observations at sea, and the second one also with refractions from land. It is understood that the refractions for the Tropics[49] which he lists in his Table I relate to 80°F and 30 inches of mercury because he shows how his values differ from such conditions. Similarly for his Table III[50] where he lists his refractions for the Temperate Zones related to 50°F and the same pressure. Converting these refractions to the conditions used in this study shows considerable differences, when compared to other results. Because those data do not distinguish between sunrise and sunset observations and due to the large differences to other results, only his refractions — *T*/*P* converted to the conditions used in this study — have been listed in Table VII. Those observations do not enter the subsequent comparisons of observed refractions.

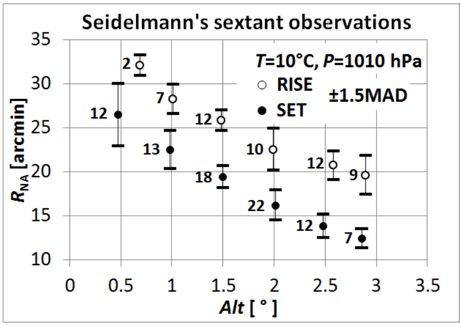
TABLE VII

Smiley's resulting refractions converted to NA and Std condition.

Alt		TROPICS				TEMP.	SEA	LAND	
[°]	N_{obs}	R_{NA} [']	$\pm\lvert\Delta\rvert$ [']	R_{Std} [']	$\pm\lvert\Delta\rvert$ [']	R_{NA} [']	R_{Std} [']	R_{NA} [']	R_{Std} [']
10	29	5·569	0·043	5·447	0·042	5·525	5·404	5·445	5·326
8	33	6·889	0·075	6·737	0·073	6·939	6·786	6·839	6·688
6	49	9·098	0·086	8·891	0·084	9·224	9·013	8·960	8·756
5	61	10·899	0·086	10·646	0·084	10·781	10·531	10·305	10·065
4	56	13·32	0·12	13·00	0·12	12·70	12·40	12·10	11·81
3	47	16·53	0·16	16·11	0·16	15·32	14·94	14·92	14·55
2·5	38	18·56	0·18	18·08	0·18	16·98	16·54	16·81	16·38
2	35	20·90	0·21	20·34	0·20	19·01	18·50	19·12	18·61
1·5	30	23·57	0·25	22·91	0·24	21·54	20·94	21·94	21·33
1	21	26·57	0·34	25·79	0·33	24·74	24·01	25·23	24·48
0·5	15	29·94	0·27	28·99	0·26	27·32	26·46	28·85	27·94
0	6	33·78	0·27	32·64	0·26				

(*vii*) Sunrise and sunset observations by Seidelmann *et al.*

Shaeffer & Liller[24] published low-altitude refractions measured by Ken Seidelmann together with three of his colleagues from USNO. They measured with a sextant the angular distance between the Sun's limb and the apparent horizon and compared those values to the Sun's true (unrefracted) position for given location and time. Their observations were only made during two consecutive sunrises and sunsets in 1968 November on the coast in North



	SunRISE			SunSET	
Alt [°]	R_{NA} [']	|Δ|[']	*Alt* [°]	R_{NA} [']	|Δ|[']
0.69	32.1	1.2	0.47	26.5	3.6
1.01	28.3	1.7	0.98	22.5	2.2
1.48	25.9	1.2	1.50	19.4	1.2
1.99	22.5	2.4	2.01	16.2	1.7
2.58	20.7	1.6	2.48	13.8	1.3
2.89	19.6	2.2	2.86	12.4	1.1

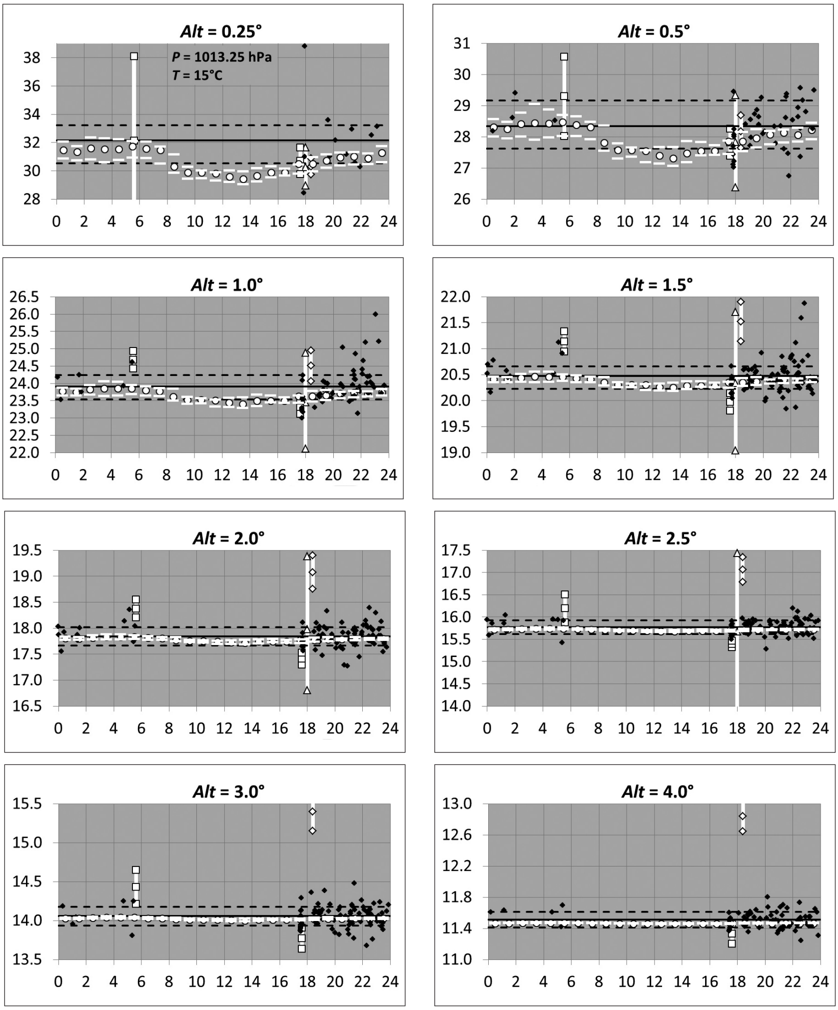
FIG. 14

Mean refractions as obtained by converting Seidelmann's observed sunrise and sunset refractions first to NA conditions and then by averaging them in statistical bins with a width of 0·5 degree altitude. The figure shows the mean and the 1·5MAD variation of each bin's content. The numbers indicate the number of observations in the bin. The text suggests a possible cause for the unexpected large difference between the sunrise and sunset refractions near *Alt* = 3°.

Carolina (Köppen Cfa). The data from their observations have been obtained by digitizing their Fig. 1 in ref. 24. Fig. 14 shows how their sunrise and sunset observations compare. Note the large refraction difference of about 7 arcmin near *Alt* = 3° between the sunrise and sunset values. At this altitude this difference is expected to be much less than 1 arcmin. A possible cause for it is probably the existence of an abnormal dip during the two days of their observations. An abnormal dip results from abnormal terrestrial refraction of the apparent horizon; this is more probable near sunrise than sunset. The difference originates from clearing the sextant observations without measuring the dip itself but rather assuming for it a standard value which is normally completely sufficient. Due to the large differences these observations have not been included in the subsequent comparisons of observed refractions.

(*viii*) Sugawa's monthly night-time refractions for Sendai, Japan.

Sugawa[25] obtained his monthly mean refractions in the altitude range 5° ≤ *Alt* ≤ 85° by ray-tracing calculations using 12 monthly mean temperature-gradient profiles which he derived from statistical analysis of 323 night-time radio-soundings made in 1948 by the Meteorological Observatory in Sendai, Japan. For calculating the refractions he used monthly mean densities caused by monthly mean temperatures and pressures. Calculating refractions with his monthly temperature-gradient profiles provides comparable results to those of this study. At his lowest altitude *Alt* = 5° one obtains for his monthly profiles an annual mean refraction at NA condition of 9·8052 ±0·0013(σ) arcmin compared to the 9·8015 ±0·0026 (1·5MAD) arcmin as obtained here for the night-time refractions. The refractions in Sugawa's Table 3 are, however, questionable. At *Alt* = 5° he obtains an annual mean refraction of 717·55 arcsec corresponding with the standard deviation of his 12 monthly values to 11·96 ±0·57 arcmin. Such a high refraction would correspond to an annual mean temperature of –36°C with monthly variations (σ) of ±10°C. This is quite unlikely for a temperate climate. Sugawa's values have therefore not been included in the subsequent comparisons of observed refractions.



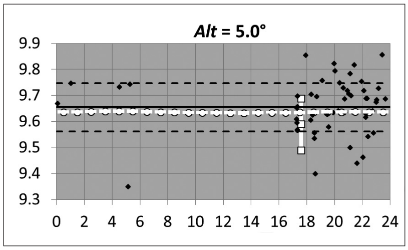


FIG. 15

The different panels show for selected altitudes how the diurnal refractions (in arcmin) from the sounding data compare with the direct measured ones from Argelander (sunset, night, and a few before sunrise, shown as black diamonds), Hellerich (sunrise and sunset, shown as white squares), Barbados (sunset, shown as white diamonds), and at Istanbul (sunset, shown as white triangles). The black lines correspond to the mean of Argelander's data (solid) and their variability range (dashed). Argelander's small number of sunrise and sunset observations compared to the night-time ones was thought to justify showing in this figure at the selected altitudes the mean and variability limits of all his observations. The variabilities correspond to the two quantile limits ⅙ and ⅚, except those from Hellerich which are 1·5MAD values and those from Barbados which are mean standard deviations. The sunrise and sunset results should all be shown close to *LHr* 6 and 18; for better visibility they were, however, distributed around these values.

9. Comparison of observed and other published LPAARs

(*i*) Comparing diurnal refractions.

In Fig. 15 the diurnal refractions from the sounding data are now compared with several of the selected directly measured ones. The quantile limits of the sounding data refractions are those as shown in Fig. 8 with white bars; the mean of their refractions (white discs) are here, however, those resulting from ray-tracing calculations using the logarithmically-corrected mean temperature gradients shown in Fig. 2. The different panels show that the night-time refractions obtained from the radio soundings tend to be within the lower quantile limit of Argelander's observations and that at the different altitudes the mean of the refractions obtained from the soundings are systematically somewhat below those from Argelander, thus indicating a marginal bias. Some of the low-altitude panels indicate that also the means of Argelander's sunset refractions agree closely with those of the soundings.

The mean values of the sounding data agree mostly remarkably well with the corresponding mean values of the sunset observations at Istanbul; only the results for $Alt = 2{\cdot}0°$ show a somewhat greater difference between the two mean values, but they are all well within the corresponding variability ranges.

The refractions from Hellerich's observations agree only partially within the variability ranges of the sounding data or Argelander's observations; his sunrise observations tend to result in larger and his sunset observations smaller refractions, with his sunrise refractions deviating somewhat more than his sunset ones. However, some of Argelander's sunrise observations agree closely with Hellerich's values.

At low altitudes the refractions from the sunset observations on Barbados agree well with those from the sounding data or those observed at Istanbul. However, with increasing altitudes those results start systematically deviating far beyond their estimated error limits; this probably originates from an inaccurate camera calibration.

(*ii*) Comparing measured and theoretical refractions at the four cardinal parts of the day.

The comparison of the LPAARs from various sources with those from this study is done in Table VIII for the four cardinal parts of the day, and with the results from the sounding data designated as $R(T')$. Table VIII has been 'built' around the reproduction of Fletcher's Table 1 which claims to show the refractions at NA conditions. Its content has been split into three parts with Table VIIIA showing the results for sunrise and daytime, Table VIIIB for sunset, and Table VIIIC for night-time. All the refractions are in minutes of arc and also their variabilities, which are generally expressed with the 1·5MAD value. Bessel's refractions listed in Fletcher's Table 1 are those which Bessel published in 1830 in his *Tabulae regiomontanae*[41]. Table VIIIC shows now that these values from Bessel agree with those resulting in this study from the reanalysis of Argelander's observations within 0·04 arcmin. It shows further that Bessel did not publish Argelander's sunset observations at $Alt < 0{\cdot}5°$ which here have been added with one exception. Argelander made at $Alt = 0{\cdot}25°$ three sunset observations where one of them is an outlier having a refraction difference of about 6 arcmin; this outlier is visible in the first panel of Fig. 15.

Fletcher provides in his 1952 publication[52] an overview of the various findings on low-altitude refractions as relevant to navigational observations. In his Table 1 he compares low-altitude refractions at NA conditions as published by Harzer, Garfinkel, Radau, Bessel, and Pulkovo; all these refractions result

TABLE VIII A

Sunrise and daytime refractions:

	A	B	C	D	E	F	G	H	I
Alt	SUNRISE							DAY	
	$R(T')$ this study sounding data		Argelander reanalysis			Hellerich 1928 measurem. Sun Ø contr.		$R(T')$ this study sounding data	
[°]	$R_{R\,NA}$	±|Δ|	*N*	$R_{R\,NA}$	±|Δ|	$R_{R\,NA}$	±|Δ|	$R_{D\,NA}$	±|Δ|
10	5·2869	1E-04						5·2867	1E-04
8	6·5160	3E-04						6·5154	4E-04
6	8·415	1E-03						8·413	2E-03
5	9·802	2E-03	2	9·71	0·30			9·797	3E-03
4	11·672	5E-03	2	11·77	0·21			11·661	6E-03
3	14·299	0·013	2	14·30	0·34	14·6	0·2	14·271	0·015
2·5	16·039	0·021	3	16·08	0·27	16·4	0·3	15·990	0·025
2	18·186	0·038	3	18·42	0·47	18·7	0·2	18·098	0·042
1·5	20·888	0·068	2	21·47	0·16	21·6	0·2	20·718	0·073
1	24·38	0·13	1	25·19		25·3	0·2	24·01	0·14
0·5	29·16	0·36				30·1	1·3	28·13	0·28
0·25	32·50	0·77				33·1	6·2	30·43	0·45
0	38·4	2·3						32·2	0·9

Col. A – G - sunRISE refractions.
A & B: Refractions from temperature gradients collected in the time-span $5 \leq LHr \leq 7$ with their 1·5 MAD variabilities.
C – E: Refractions from this study's reanalysis of Argelander's star observations. A few of his star observations were done shortly before sunrise between $5 \leq LHr \leq 6$.
F & G: Refractions as derived in this study from Hellerich's measured vertical contraction of the Sun's diameter; and their 1·5 MAD variability relative to the fitted Bennett-type refraction function.
Col. H & I - DAYtime refractions.
Refractions as obtained with the sounding data for the time span ($9 \leq LHr \leq 17$). It appears that no daytime astronomical refractions have been published before which would allow a comparison.

from theoretical considerations, except those from Bessel which are claimed to be derived from observations by Bradley and below 5° altitude by Argelander. Radau's original refractions have later been corrected using a slightly different refraction constant; his table lists these values under Radau (mod.). This study finds that the theoretical refractions which are generally based on a constant temperature gradient compare best with the conditions near sunset. Bessel's refractions were however mostly measured during the night.

(*iii*) Comparing the seasonal refractions.

The sunrise and sunset observations at Edmonton and the sunset observations at Istanbul allow comparison of observed seasonal refractions with those from the sounding data. The results for Edmonton are, however, from a completely different climate. The results from the comparison are shown in Fig. 16. The middle of the three values shown for each season represents the mean and the outer ones either the ⅙ and the ⅚ quantile limits or, as for Edmonton, the standard deviation. The mean $R(T')$ values are calculated with log-corrected profiles.

Sampson[12] lists in his Table 3.4.1 the monthly mean refractions and their standard deviations. For comparing his results with the six seasons, as used in this study, his monthly mean values and their standard deviations have

TABLE VIII B

Sunset refractions:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
Alt	← SUNSET →																	
	$R(T')$ this study sounding data		Argelander reanalysis			Istanbul this study measurem.		Barbados Köppen Aw measurem.		Hellerich 1928 measurem. Sun Ø contr.		Theoretical $R_{S\,NA}$ values from …						
												Garfinkel 1967		Fletcher				
[°]	R_{SNA}	$\pm\|\Delta\|$	N	$R_{S\,NA}$	$\pm\|\Delta\|$	R_{SNA}	$\pm\|\Delta\|$	R_{SNA}	$\pm\sigma$	R_{SNA}	$\pm\|\Delta\|$	his Tab. IV		Garfinkel 1944	Pulkovo	Radau	Radau (mod.)	Harzer
10	5·2865	1E-04												5·3	5·3	5·3	5·3	5·3
8	6·5147	5E-04												6·6	6·5	6·5	6·5	6·5
6	8·411	2E-03												8·5	8·4	8·4	8·4	8·4
5	9·793	3E-03	15	9·79	0·09			11.3	0.2	9.6	0.1	9·8	9·8	9·9	9·8	9·8	9·8	9·8
4	11·653	7E-03	20	11·69	0·10			13·1	0·2	11·4	0·1	11·7	11·7	11·7	11·7	11·7	11·7	11·7
3	14·253	0·018	26	14·32	0·12			15·7	0·3	13·9	0·1	14·3	14·3	14·4	14·3	14·4	14·3	14·3
2·5	15·964	0·029	24	16·04	0·15	16·0	2·1	17·5	0·3	15·6	0·1							
2	18·059	0·050	22	18·17	0·17	18·4	1·6	19·5	0·3	17·6	0·1	18·1	18·1	18·3	18·2(1)	18·3	18·2	18·1
1·5	20·658	0·085	21	20·81	0·17	20·9	1·6	22·1	0·4	20·3	0·2			20·9	20·8	20·9	20·8	
1	23·93	0·16	14	24·05	0·30	24·0	1·6	25·2	0·5	23·8	0·2	24·0	24·1	24·3	24·2	24·3	24·2	24·2
0·5	28·08	0·32	10	28·57	0·74	28·6	1·5	29·0	0·5	28·5	0·4			28·7	28·6(5)	28·7	28·6	
0·25	30·56	0·54	*2*	*30·5*	*2.0*	31·2	1·4	31·2	0·6	31·5	1·0							
0	33·3	1·2	2	34·3	0·6	34·5	2·7	33.7	0.6			33·6	33·9	34·4	34·3	34·5	34·3	34·5

A & B: Refractions from temperature gradients collected in the time-span 17 ≤ *LHr* ≤ 19 with their 1·5 MAD variabilities.
C – E: Refractions and their 1·5 MAD variabilities from the reanalysis of Argelander's sunset observations. The values at $Alt = 0{\cdot}25°$ are in italic because from the initial three observations an outlier with about 6 arcmin difference has been removed.
F & G: Istanbul refractions as obtained from measuring the Sun's limb positions above the sea horizon; note the 1·5 MAD variability contains also contributions from the dip.
H & I: Refractions from the sunset observations at Barbados by Sampson *et al.*[14] when assuming average temperatures of 27°C and average pressure of 1010 hPa. The standard deviation contains also contributions from the dip.
J & K: Refractions as derived here from Hellerich's measured vertical contraction of the Sun's diameter; and their 1·5 MAD variability relative to the fitted Bennett-type refraction function.
L – R: Theoretical refraction estimates. The theoretical models tend using a constant temperature gradient in the troposphere which agrees best with the situation near sunset.
L & M: Garfinkel's published[16] in his Table IV refraction results from six different temperature gradient and refractivity combinations. The values in the two columns show the refractions from the two most extreme cases.
N – R: Fletcher[52] published refractions from different researchers; here only the theoretically derived ones are shown. Bessel's observed refractions, which make also part of Fletcher's table, are shown in Table VIII C with the night-time refractions.

TABLE VIII C

Night-time refractions:

	A	B	C	D	E	F	G	H	I	J	K	L
Alt	◄ NIGHT ►											
	$R(T')$ this study sounding data		Argelander reanalysis			Historical refractions						
						Cassini 17th c.	Bradley 1750?	Bessel 1755	Laplace 1805	Bradley 1812	Bessel 1830	Main 1857
[°]	$R_{N\ NA}$	$\pm\|\Delta\|$	*N*	$R_{N\ NA}$	$\pm\|\Delta\|$	$R_{N\ NA}$						
10	5·2869	1E-04				5·51	5·28	5·28	5·32	5·29	5·30	5·30
8	6·5160	3E-04				6·93	6·53	6·50	6·56	6·54	6·53	6·51
6	8·415	1E-03				9·31	8·47	8·40	8·48	8·50	8·43	8·41
5	9·801	3E-03	47	9·83	0·10	~~11·25~~	9·86	9·79	9·88	9·95	9·79	9·78
4	11·671	5E-03	56	11·73	0·13	~~14·24~~	11·70	11·67	11·77		11·70	11·65
3	14·297	0·013	76	14·33	0·12	~~19·49~~	14·02	14·33	14·43		14·31	14·31
2·5	16·035	0·022	75	16·10	0·17	~~24·06~~	~~15·27~~	16·11	16·18		16·09	16·09
2	18·179	0·039	81	18·23	0·19	~~31·90~~	~~16·31~~	18·32	18·32		18·23	18·29
1·5	20·875	0·071	82	20·94	0·23	~~51·76~~	~~16·54~~	21·11	20·97		20·94	21·08
1	24·35	0·15	55	24·57	0·43		~~14·72~~	24·74	24·29		24·52	24·71
0·5	29·08	0·38	34	29·20	0·54		~~9·20~~	29·58	28·45		29·18	29·54
0·25	32·33	0·76	9	32·79	1·6		~~4·87~~	32·66	30·94			32·61
0	37·8	2·2						36·26	33·66			36·21

A & B: Refractions calculated with temperature gradients in the time-span 23 ≤ *LHr* ≤ 7+24 with their 1·5 MAD variabilities.
C – E: Refractions from this study's reanalysis of Argelander's star observations.
F – L: Historical refractions: It is assumed that these refractions were originally intended to be used for astronomical observations during the night. Cassini's 17th Century theoretical model (in col. F), as described in[53], provides reasonable refraction estimates as low as, *e.g.*, 6° altitude. Bradley (col. G & J) performed his measurements around 1750 at the Royal Observatory but all of his work has only been published after his death in 1762. Col. G contains Bradley's refraction values resulting from the following empirical function, valid for $P = 29{\cdot}6$ inches of mercury and $T = 50°$F but here converted to NA conditions which — according to Maskelyne[3] — Bradley found around 1750 as an improvement to a previous empirical estimate function:

$$R[\,°\,] = 57/3600 \cdot \tan[ZD - 3 \cdot 57/3600 \cdot \tan(ZD)] \tag{7}$$

This improved estimate function could however still not be used beyond *ZD* of about 85°. Bessel (col. H) based in his *Fundamenta*[7] the refraction values on those of Bradley but adjusted them according to his own findings (for more details see, *e.g.*, ref. 9). (The value for $Alt = 0{\cdot}25°$ has been obtained by quadratic interpolation using his published four values within 0° ≤ *Alt* ≤ 1°.) Col. I shows the refractions which, according to ref. 54, p. 341 and 342, were calculated with Laplace's theoretically derived formulae published in his *Mécanique Céleste,* ref. 55, pages 264 and 271. Note that his refractions relate best with other theoretically derived refractions as listed in VIII B col. L–R for sunset conditions. Col. J shows how Bradley's refractions, some decades after his first published ones (col. G), have been further adjusted[56]. When realizing that their empirically derived formulae for estimating the refraction (in col. H) would not be useful at low altitudes, Argelander measured refractions below 5° altitude. Those results entered then Bessel's *Tabulae Regimontanae*[41] (as shown in col. K). Rev. Main[9] (col. L) proposed to apply some small improvements to Bessel's refractions as published in the *Fundamenta* (col. H) and the *Tabulae* (col. K); based on his own observations performed between 1836 and 1854 at the Royal Observatory and using also observations made at the Cape of Good Hope Observatory he suggests: "On the whole, I should recommend, … for large zenith distances, that the mean refractions of the *Tabulae* be used without alterations as far as 82°; that from 82° to 85° the logarithms of the mean refractions of the *Tabulae* be diminished by 0·00100; and, finally, that from 85° to 90°, the refractions of the *Fundamenta* be diminished in the proportion of 1: 0·99848, …".

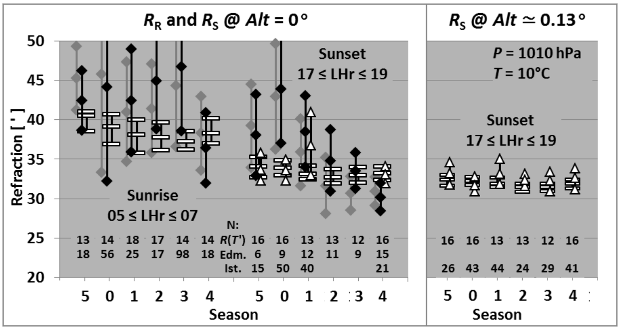


FIG. 16

Comparison of seasonal refraction variabilities: the results from the sounding data are represented with white bars, those from the Istanbul observations with white triangles, and those from Edmonton with black and grey diamonds; with the grey ones relating to the original, non-T/P-converted values. The small numbers above the x-axis indicate the number of data in the various sets.

been pooled. For converting his refractions to some reference conditions the following estimates were made by visual examination of the point clouds in his figures. From Fig. 3.5.4 and 3.5.5 a mean pressure for all seasons of 935 hPa has been selected. The average sunrise temperatures for the seasons 0 to 5 have been estimated with his Fig. 3.5.13 in °C as –8, +2, +9, +5, –10, –12, respectively, and with his Fig. 3.5.14 for the sunset temperatures in °C as +0, +8, +12, +11, –7, –8, respectively. His seasonal variabilities are greater than those from this study; one reason for it is because his refractions relate to the atmospheric conditions during the observations and therefore contain also the seasonal temperature and pressure variations. The systematically somewhat greater average refractions at sunrise and sunset have already been discussed in 8*iii*.

The Istanbul observations allowed observing the setting Sun down to the apparent sea horizon only from the first half of September to the first half of April. During the remaining part of the year the sunsets were at an azimuth where the skyline of the European part of Istanbul protruded from behind the apparent sea horizon up to several arcmin above the astronomical horizon. It is for this reason that in the right panel of Fig. 16 the Istanbul observations have additionally also been compared with those from the sounding data at $Alt \simeq 0{\cdot}13°$. For this the refractions and the quantile limits from the sounding data which were calculated for $Alt = 0{\cdot}10°$ and $0{\cdot}15°$ have been linearly interpolated to the slightly different mean altitudes of the Istanbul observations which are 0·12°, 0·11°, 0·15°, 0·16°, 0·12°, and 0·12° for the seasons 0 to 5 correspondingly.

Fig. 16 shows that the sunset refractions at Istanbul generally compare well with those obtained from the sounding data. Only the one from Early Spring (1_wSpr) shows a greater difference and a considerably larger variability. Because the refractions from the sounding data do not show this effect, one can conclude that the increased difference probably originates, at least partially, from abnormal terrestrial refraction at the apparent horizon (dip). This can be explained with the increased occurrence of considerably warmer air flowing over the still-cold sea surface during Early Spring which can cause strong inversions, and thus lead to abnormal *terrestrial* refraction of the dip. When neglecting this seasonal outlier, then the average refraction difference between the Istanbul observations and the results from the sounding data is at $Alt = 0°$ –0·2 arcmin, and at $Alt \simeq 0{\cdot}13°$ +0·2 arcmin. The variabilities for the individual Istanbul observations are roughly 1·5 to 2 times larger than those from the sounding data. This ratio agrees with the findings discussed in (*iv*) below. The Edmonton data indicate also such a seasonal outlier with a considerably larger variability. There the inversions appear to occur somewhat earlier in mid-Winter (0_Wint). The author cannot explain why at Edmonton the sunrise refractions are during Late Spring and High Summer maximum whereas for the refractions from the sounding data they are minimum.

(*iv*) Comparing refraction variabilities.

Fig. 17.1 shows the relative and absolute refraction variabilities from the various observational data discussed in this study. The relative night-time variabilities in panel *b*1 show that Argelander's measurements at $Alt \gtrsim 2°$ have a fairly constant value of about 1%, which appears to reflect his instrument's accuracy. The results from the sounding data converge differently: with increasing altitudes they decrease towards zero without reaching such a limiting value. With this decreasing variability at increasing altitudes the refractions derived from the radiosonde data thus reflect Oriani's theorem[4], *i.e.*, that the refraction at higher altitudes depends almost exclusively on the air pressure and temperature at the observer's eyes.

The sunset observations at Istanbul which were observed from an average height of eye around 3·5 m show a minimal variability near $Alt = 0{\cdot}5°$ with

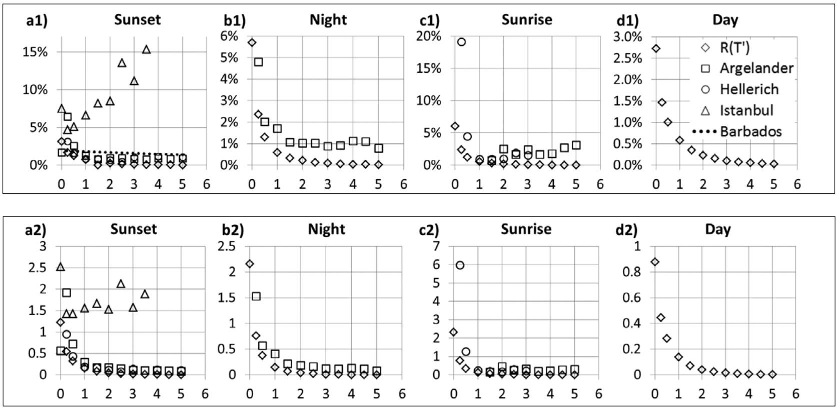


FIG. 17.1

Comparison of the relative and absolute refraction variability (1·5MAD, and for Barbados σ) resulting from the sounding data, $R(T')$, with those of various directly observed ones for the four cardinal parts of the day. The upper row shows the relative refraction variability in %, as a function of altitude, *Alt* [°] and the lower row the absolute refraction variability in arcmin as a function of *Alt* [°]. The various results are discussed in the text.

increasing values below and above that altitude. Those above are attributed to the increased Sun brightness which makes locating the limb position in the overexposed Sun's image more difficult. The higher variabilities below likely reflect the uncertainties in locating the exact apparent horizon and from varying surface roughness[57], *i.e.*, the varying estimated wave heights at the horizon. This effect is particularly important for an observer close to the surface for whom the astronomical horizon is seen when looking through the lowest part of the logarithmic temperature-gradient profile, *i.e.*, where small deviations cause maximum differences.

The refraction variabilities of the Barbados observations correspond to Sampson's fitted mean, *i.e.*, of his eq. 4. The very low values are possibly a result from very stable atmospheric conditions in this wet tropical climate. Because his variabilities do not account at higher altitudes for the increasing bias of his observed refractions, which reaches at $Alt = 4°$ about 1·3 arcmin (see corresponding panel in Fig. 15), his absolute variabilities are not shown in panel *a*2.

The data points in panel *b*1 at $Alt \leq 1°$ suggest that the variability of Argelander's individual measurements are roughly twice as large as those resulting from the radiosonde data where one 'individual' result is obtained from the mean of at least ten soundings. Fig. 17.2 shows how this ratio increases at higher altitudes. At $Alt \lesssim 0{\cdot}5°$ the factor is on average about 1·6. Considering that Argelander's observations at $HoE = 50$ m scatter expectedly somewhat less than they would at $HoE = 2$ m, then it appears safe using near the astronomical horizon at the lower height a factor of 2 instead. This means that the variabilities designated above as the 'meteorological', 'seasonal', or 'local' are near the horizon for individual observations in reality about twice as large as those derived from the sounding data. Applying this ratio to the absolute variability for the radiosonde-derived sunset data at $Alt = 0°$ in Fig. 17.1 panel *a*2, then the ±1·2 arcmin increase for individual sunset observations to about ±2·4 arcmin which compares well with the ±2·5 arcmin variability of the Istanbul observations containing additionally the variability of the dip's terrestrial refraction. In panel *c*2 Hellerich's large ±6 arcmin variability for his few sunrise observations near the horizon becomes plausible when applying the factor of 2 to the ±2·3 arcmin from the soundings.

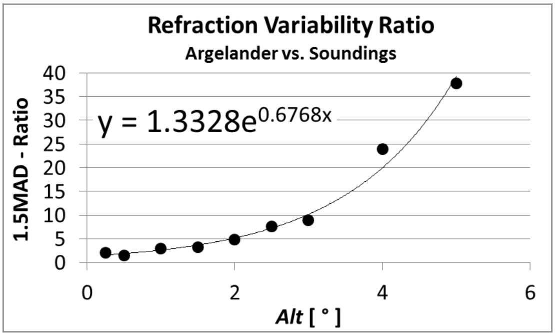


FIG. 17.2

Ratio indicating the factor by how much the refraction variabilities from the soundings have to be increased to correspond with the variabilities from Argelander's individual observations. The individual ratios relate to the 1·5MAD values for the night-time refractions in Table 7.2.

10. Concluding remarks

It has been found that the refractions calculated with the statistically mean temperature-gradient profiles derived from radio-sounding data, collected at different locations during the same local hour, compare well with directly measured refractions and can well describe their diurnal cycle. Previously LPAARs were measured at night when refractions are high. Calculated refractions, however, which for all of the troposphere generally used some constant 'average' temperature gradient, agree best with the conditions near sunset where refractions are somewhat smaller than at night. Here it has been found that during the day the LPAARs are smallest with a minimum shortly after the Sun's transit. The daytime refractions could not be compared with measurements; the agreement obtained with various direct measurements suggests, however, that these results appear realistic. The resulting values for the four cardinal parts of the day — as applicable for coastal locations in a temperate climate — have been listed in Table VIII. The *mean*-refraction at the astronomical horizon for $P = 1010$ hPa and $T = 10$°C is during night-times 37·8 ±2·2 arcmin, near sunrise 38·4 ±2·3 arcmin, during daytimes 32·2 ±0·9 arcmin, and near sunset 33·3 ±1·2 arcmin. The 'seasonal' variability contribution was estimated to be less than about ±0·35 arcmin, and the 'local' one less than about ±0·24 arcmin when freed from temperature and pressure variations. Comparing the variabilities with those of individual refraction observations indicate that those resulting from the sounding data have, near the astronomical horizon, to be increased by a factor of about 2 in order to become comparable with individual observations; this factor increases with higher altitudes. This means that for an observer near sea-level the expected day-to-day variabilities for individual refraction observations are at the astronomical horizon during the night ±4·3, during sunrise ±4·6, during daytime ±1·8, and during sunset ±2·4 arcmin; the 'seasonal' variability is then expected to be maximum about ±0·7 arcmin and the 'local' one maximum about ±0·5 arcmin.

It is shown that the conversion of refraction to different atmospheric conditions depends not only on the parameter in question (temperature or pressure) but also on the *HoE* and the observed altitude. In this context it was also shown how the refraction's dependency on temperature changes from being inversely proportional at higher altitudes, as used by astronomers, towards inversely squared proportional when approaching the horizon, as used by surveyors.

The results from this study do actually confirm Bouguer's rule[23] from 1749 that during the day refractions would be ⅙ to ⅐ lower than at night; the ratio found here is $1/6{\cdot}7$ but it applies only at $Alt = 0°$ — the denominator increases rapidly at higher altitudes; at $Alt = 0{\cdot}1°$ the ratio is already $1/10{\cdot}4$.

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Appendix — List of variables and abbreviations:

Variable	*Description*
Alt or Alt_{app}	Apparent (refracted) altitude of astronomical object as observed at *HoE*.
Alt_t or Alt_{true}	Unrefracted or air-free altitude of astronomical object at *HoE*.
D	The horizon's apparent depression, apparent dip.
D_g	Geometrical (unrefracted) dip.
f_{corr}	Correction function for improving the P/T conversion; see eqns. 3.3 and 4.3.
H_{lower}	Height at lower level of a layer.
H_{upper}	Height at upper level of a layer.
HS	Angular height of the Sun's limb above the apparent horizon.
HoE	Height of eye above the Earth's surface.
MAD	Median absolute deviation (MAD); for normally distributed data: $1{\cdot}5{\cdot}\mathrm{MAD} \simeq \sigma \approx 0{\cdot}5{\cdot}\lvert Q(1/6) - Q(5/6) \rvert$
N	Number of data.
$n-1$	Refractivity of air.
Q(⅙) and Q(⅚)	Quantile limits ⅙ and ⅚.
r_c	Earth's radius of curvature at surface.
r_s	Radius of the visible Sun image.
R	Astronomical apparent refraction (relative to the object's apparent altitude, *Alt*). Note, that all refractions relate to astronomical ones, except where explicitly stated different.
R or R_{Std}	Apparent refraction at standard conditions (15°C, 1013·25 hPa)
R_{calc}	Apparent refraction calculated by ray-tracing using a Tgrad-profile.
R_{corr}	Apparent refraction calculated with the logarithmically-corrected temperature gradient in the lowest layer.
R_D	Apparent refraction during daytime.
R_{est1}	Apparent refraction estimated at different atmospheric conditions by P/T conversion using only parameter ratio η.
R_{est2}	Apparent refraction estimated at different atmospheric conditions by P/T conversion using parameter ratio η and correction factor f_{corr}.
R_h	Mean refraction for local hour, 'h'; it results from the mean of those locations contributing with any of the possible six seasons to this time interval.
R.H.	Relative humidity.
R_j	Apparent refraction calculated with the mean temperature gradient T'_j.
R_L	Apparent refraction per arc length of the ray's trajectory through the atmosphere.
R_N	Apparent refraction during night-time.
R_{NA}	Apparent refraction at Nautical Almanac conditions (10°C, 1010 hPa).
R_o or R_{lsh}	Basic apparent refractions resulting from ray-tracing calculation using basic temperature-gradient profile T'_o or T'_{lsh}.

R_{obs}	Observed refraction.
R_{orig}	Apparent refraction calculated with the statistically derived temperature gradient in the lowest layer, *i.e.*, where its value has not been replaced with the log.-corrected one.
R_R	Apparent refraction during sunrise.
R_S	Apparent refraction during sunset.
R_s	Apparent refraction for a season.
R_T	Terrestrial refraction, *e.g.*, between observer and the horizon.
T'_h	Temperature gradient for a certain local hour (*LHr*); it results from the mean of those locations contributing with any of the possible six seasons to this time interval.
T'_j	Mean temperature gradient from all the selected T'_o within the jack-knifed set.
T'_o or T'_{lsh}	Basic temperature-gradient profile where location '*l*', season '*s*', and local hour (*LHr*) '*h*' are identical.
T'_s	Temperature gradient for a certain season; it results from the mean of those locations contributing with any of their soundings (local hours) to it.
ZD	Zenith distance.
¦Δ¦	Probable error.
$\|\Delta\|$	1·5 MAD.
λ	Wavelength.
σ	Standard deviation.

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CORRESPONDENCE

To the Editors of 'The Observatory'

A Final Salvo on the Velocity of Gravitational Waves

My thanks once again to Jonathan Thornburg[1], and now to Philip Helbig[2], for their detailed replies to my further letter on this topic[3]. I am grateful to them both for taking this discussion seriously even if, indeed perhaps especially as, we don't entirely agree at some points of the argument, and hope that other readers of *The Observatory* may have found the result as interesting as I have. One thing, at least, which this correspondence seems to confirm is my original contention that the subject of 'the why and the wherefore' of gravitational and electromagnetic waves propagating at the same velocity in free space is far from straightforward in the existing literature!

Undoubtedly the subject could be pursued further, probably very much further, but a detailed point-by-point response here to Thornburg's and Helbig's recent letters is beyond my current appetite for the subject and probably also that of the Editors. I am entirely open, however, to the possibility of a further consideration of the points raised in those most recent communications requiring a modification of the position set out in my previous letters. That would certainly be welcome as a learning exercise and a clarification.

As a last thought from me on the subject, on which perhaps we may all nevertheless be able to agree, the following further classical-physics argument is offered: (*i*) Although the full mathematical analysis of wave-propagation in General Relativity is somewhat prolix and its physical interpretation was for many years uncertain, the final result of the theory is simple enough: *Waves carrying real energy through a weak gravitational field propagate at the fundamental velocity incorporated in the metric when constructing the theory*. That much is straightforward, even if its rigorous derivation is not, and for clarity's sake certainly deserves stating as a formal theorem of GR; (*ii*) GR contains Special Relativity as a particular case, therefore consistency demands that the fundamental velocity in GR is that of the Minkowski metric in the special theory; (*iii*) The uniqueness argument of my original letter[4] — which still remains valid so far as it goes, even though we are all agreed it is not sufficient on its own — now shows that that fundamental velocity can *only* be 'c'. Q.E.D.

At the very least, surely this provides a good outline of the reason for $c^\star = c$ in teaching the subject? And I can hardly accept that simplicity of derivation is a purely subjective matter in the eye of the beholder as, in that case, there would be no need for Complexity Theory in numerical computation — recall the famous case in solving large linear systems, of reduction to triangular form *versus* use of determinants — and broadly similar considerations are obviously applicable to more abstract symbolic analysis (just think of Wiles' 1993/5 proof of 'Fermat' compared, say, with Euclid's of the infinity of primes).

Yours faithfully,
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This correspondence is now closed.

REVIEWS

Time: From Earth Rotation to Atomic Physics, 2nd Edition, by Dennis D. McCarthy & P. Kenneth Seidelmann (Cambridge University Press), 2018. Pp. 386, 25·5 × 18 cm. Price £59·99/$84·99 (hardbound; ISBN 978 1 107 19728 2).

Have you ever wondered, when you stand astride the brass marker on the zero meridian of longitude at the Royal Observatory, Greenwich, why your GPS receiver says you are 102 metres west? Why do we add 1 second to our clocks at midnight at the end of some years, or at the end of June in others? Why don't we subtract 1 second sometimes, instead?

You will find the answers to these and many more questions in this excellent book, written by two experts who worked on the practical aspects of these topics at the US Naval Observatory. Their reference book covers the historical development of the measurement and distribution of time, from sundials to atomic clocks. It explains the establishment of the stellar reference frame against which the Earth's variable rotation is measured, and the development, definition, and maintenance of the reference frame on the Earth's surface. The development of techniques used to measure the Earth's rotation, from ancient reports of eclipses to counting pulses from pulsars, is discussed.

This second edition brings these subjects right up to date, and investigates the possible future developments in timekeeping. These include the thorny question of whether to let our clocks, regulated by an atomic frequency, drift away from the traditional time kept by our clocks based on the period of the Earth's rotation relative to the Sun. Does it matter? Consult this book and find out. — L.V. MORRISON.

Time and Time Again: Determination of Longitude at Sea in the 17th Century, by Richard de Grijs (Institute of Physics Publishing), 2018. Pp. 307, 26 × 18·5 cm. Price £99/$150 (hardbound; ISBN 978 0 7503 1195 3).

For people afloat on the Earth's seas and oceans one of the frequently asked questions was "where are we?". Two numbers were needed; latitude and longitude. The first was relatively easy to find, the elevation of the pole star or the noon-day Sun providing a useful datum. Longitude was the problem.

Most of us are familiar with both Dava Sobel's 1995 best-seller, *Longitude*, and John Harrison's Greenwich chronometers (starting with H1 in 1736), so the next question is "what does de Grijs add to the discussion?" Well the clue is in the

"17th Century" in the book's title. It really should be "up to the 17th Century" because de Grijs takes us back to the very start of sea travel considering the problems the Greeks had in finding their way around the Mediterranean Sea. He then reviews the steady increase in the accuracy of cartography and the spur provided by the Islamic requirement of knowing the direction of Mecca.

Ocean navigation advanced considerably with the 'discovery' of North America by Christopher Columbus and the rivalry between Spanish and Portuguese explorers. Much is made of the endeavours of the Dutch East India Company. De Grijs then discusses the two major longitude-measuring proposals. First we had the astronomical approach using timings of the eclipses of the satellites of Jupiter, or the accurate measurement of the movement of the Moon against the starry background. Then there was the clock method. That was first proposed in 1530 by the Dutch astronomer Jemma Reinerszoon, but required a clock that kept accurate time throughout a long journey across stormy oceans. The challenges of high-precision timekeeping are considered in great detail, and terms like clock remontoires and the isochronism of cycloidal pendula spring from the page. The horological approach of Galileo, Christiaan Huygens, Robert Hooke, and Isaac Newton are thoroughly investigated. The book ends with the introduction of the 1714 Longitude Act and the establishment of a prime meridian. So essentially the book ends where Sobel starts.

De Grijs is a professor at the Kavli Institute of Astronomy and Astrophysics at Peking University, China. What I liked especially about this book was its thorough academic approach, its clarity, and the depth of knowledge and understanding portrayed by the author. If a topic required mathematics, it got mathematics; if specific letters were important, they were quoted in full; if a diagram was significant, it was included. And hundreds of references were added so that you could check things for yourself. My only criticism is that someone seems to have left out the index. — DAVID W. HUGHES.

Five Photons: Remarkable Journeys of Light across Space and Time, by James Geach (Reaktion), 2018. Pp. 184, 22 × 14·5 cm. Price £14·95 (hardbound; ISBN 978 1 78023 991 0).

I like the title of this book. Nearby pubs include *The Three Pigeons* and *The Seven Stars*, and I feel sure that *The Five Photons* would be well patronized, with catchy names like 'Time Dilation' and the 'Ultraviolet Catastrophe' for its House Ales, and questions about quantum mechanics on quiz nights (the answers to which will be both right and wrong). I also like its compact size and the idea behind the title of providing the reader with a broad cosmic overview by describing in succession the information obtained from different light sources across the Universe.

The book is intended for the general reader and is written in a colloquial style which is easy to follow, and it comes across to me as being right up to date in describing current cosmological awareness. It is in six chapters, the first telling the reader what light is, or in other words introducing us to the photon; the fact that the photon can be perceived as both an electromagnetic wave and a particle is well made, but inevitably and perhaps rightly sets the note that science or at least astrophysics is strange and readers will have to keep their wits about them. The next chapter, headed 'Old Light', is about what has been learned from the study of photons coming from the Cosmic Microwave Background. There is plenty of information about the Big Bang, described — rather inelegantly in my view — as when "the whole shebang was all in the same spot", and what came afterwards, including the initial ionized plasma imprisoning the photons, the general cooling which led to the formation of hydrogen, and then the Surface of Last Scattering (a phrase so Tolkienesque that I expected a subsequent reference to the Fall of Gondolin).

Subsequent chapters are headed 'Starlight', 'Dark Energy's Imprint', 'Black Hole Beacons', and 'Radio from the Cosmic Dawn', again following the basic theme of what we learn from photons and how we do it. There is appropriate mention of black holes, dark matter, dark energy, and of course space–time, and more memorable phrases including a humorous reference to the "Integrated Sachs–Wolfe Effect" (p. 100), the disarming admission that "astrophysicists have a rich history of poorly chosen nomenclature" (p. 46), and the almost Germanic "amplitude of the cosmic microwave background temperature anisotropy power spectrum" (p. 155).

In summary this book leaves the interested layman well briefed about current state-of-the-art astrophysics and emphasizes the importance and startling sophistication of current spectroscopy in obtaining information from photons, including — despite the emphasis on light — those whose electromagnetic energy produces waves outside the visible light zone. Because most photons come from far away and long ago, much of the story relates to the early Universe and the interactions of photons and fermions. Only after the Epoch of Recombination was the cosmic microwave background able to operate, and this, together with the Epoch of Re-ionization after the first stars were formed and the continuous expansion of space across which the photons must travel and the complication of redshifts, is all well described by the author. Curiously, while the fermion is mentioned, the book makes no mention of the photon as a boson. This may be due to a commendable desire to keep things simple; but simplicity is something of a lost cause in modern astrophysics, involving as it does arcane matters such as relativity and quantum mechanics.

Despite his reticence about bosons, the author has provided a copious amount of information about the Universe hinged upon photons and spectroscopy and has done so in a style which is both easy and enjoyable to read. The book has a good index, which is important because, as has already been hinted at, the author uses a lot of astrophysical terms which may be unfamiliar to many laymen (anisotropies, hyperfine splitting, triple-alpha process, *etc.*) but thanks to the index can be rapidly revisited. The book also has thirty illustrations in black-and-white diagrammatic form, which help to clarify some of the concepts described but which I found a little disappointing, because I think a book of this quality deserves some vivid use of colour in its illustrations — particularly as its hero is light.

In conclusion, I cannot resist pointing out that Dr. Samuel Johnson, who, when asked to define poetry, famously replied "Sir, it is much easier to say what it is not. We all *know* what light is but it is not easy to *tell* what it is", would have found this book very helpful. — COLIN COOKE.

The Exoplanet Handbook, 2nd Edition, by Michael Perryman (Cambridge University Press), 2018. Pp. 952, 25 × 19 cm. Price £62·99/$84·99 (hardbound; ISBN 978 1 108 41977 2).

This is a new edition of Perryman's excellent *Exoplanet Handbook* which was published in 2011. The first edition quickly became a favourite with researchers; in my view this version will become the bible. The second edition is the same format as the original but contains more than twice as many pages and this reflects the relative explosion of new results and developments in this flourishing research field. *The Exoplanet Handbook* (either edition) is a reference work — it is not a tome for casual reading, but instead is the first port of call when thinking about a new project. Perryman has painstakingly researched all the discovery and analysis techniques commonly used and presented his conclusions in a concise way whilst retaining sufficient detail to make them useful for research scientists. I don't know any other single researcher who could have compiled a similar work.

While the chapters retain the same titles in both editions, even a casual glance at the table of contents shows that the new edition is greatly enlarged. Probably the most extended is that concerned with transits, where the last eight years of results includes most of the ground-based surveys and their results for gas giants and, of course, the transformational *Kepler* results. Perryman neatly summarizes the most important discoveries and results from those surveys. Beyond transits and in this period we have also seen the first results from the AO imagers *SPHERE* and *GPI*, *ALMA*, and more radial-velocity and microlensing results. The breadth of work and Perryman's ability to digest the results and put it in context is just remarkable.

So is this the last edition of the *Handbook*? Already there is a rival version with a similar title. However, to me, at least, Perryman will always be associated with *Gaia* and these results are still to come. I wouldn't bet against Perryman producing the third edition of *The Handbook* around 2024 with, potentially, 2000 pages including the latest results from *TESS*, *JWST*, *Gaia*, *ESPRESSO*, and a number of IR spectrographs currently commissioning (no-doubt enlarging the sections concerned with life and habitability) and looking onward to results from ESA's *PLATO* and *ARIEL* missions. I'm looking forward to it already. — DON POLLACCO.

The Astronomers' Magic Envelope: An Introduction to Astrophysics Emphasizing General Principles and Orders of Magnitude, by Prasenjit Saha & Paul A. Taylor (Oxford University Press), 2018. Pp. 121, 25 × 17 cm. Price £19·99 (paperback: ISBN 978 0 19 881647 8), £39·99 (hardbound; ISBN 978 0 19 881646 1).

This is a delightful small book that is not quite what it seems at first. The title suggests (to me, at least) that it will be rather general, offering rather hand-waving estimates of sizes of astronomical objects and the like, but in fact it is firmly rooted in physics, and the readership that will get most out of this book will be students who have a good understanding of dynamics, relativity, vector calculus, and so on. The book very nicely shows what one can do, sometimes approximately, if one does have a grasp of the underlying physics. As a result, it is not aimed at first- and second-year undergraduates, but more at students and physicists with more experience — I suspect that some readers may do a bit of revision of undergraduate-physics lectures in order to make the best of the book. It contains applications in a number of astrophysical and cosmological contexts, and is bang up to date by including some material on gravitational waves from orbiting binary stars. It will be a good resource for lecturers, showing some nice examples of applications of the physical principles, and of especial interest to advanced undergraduates and postgraduates. The reader is encouraged to write computer codes to find solutions, and the student with a thirst to learn and the motivation to engage fully with the book will be amply rewarded. — ALAN HEAVENS.

Star Tales, Revised and Expanded Edition, by Ian Ridpath (Lutterworth Press), 2018. Pp. 223, 23·5 × 15·5 cm. Price £19·50/$39 (paperback; ISBN 978 0 7188 9478 8).

For many millennia, people (and perhaps animals) must have stared at the moonless night sky in wonder and in amazement at all those points of light wheeling slowly across the sky. Some of them must have speculated on what they were, and begun to make up stories to explain them, which they then passed down through the generations. With almost no global contact between people, different stories emerged from different cultures, and some of them have survived to the present day. Different cultures also invented different patterns in the sky to match their stories, and constellation patterns in different cultures differed considerably.

However, as cultures gradually came into contact, and the patterns in the sky became used by farmers to mark the seasons and then by sailors and explorers for navigation, the patterns relating to Greek and Roman myths became more widely used, and when the constellations were systematized by the IAU it was these European patterns that were used to define the constellations. Ian Ridpath's stories and legends are therefore largely those drawing on Greek and Roman mythology. Wisely, he does not try to extend his book to cover the very different stories from other ancient cultures, fascinating though they are; it would be good if someone had the skill and energy to compile an account in English of at least some of them, from Egypt, India, Australia, and elsewhere, but they are not discussed in this book.

So, what *is* discussed? The first two chapters are introductory to the main text, and indicate in particular the considerable range of sources that he has consulted: throughout, he has consulted original sources wherever possible rather than later compilations, and he lists his sources fully. In Chapter 1, Ridpath introduces the constellations, and gives a brief outline of how it is possible to estimate the latitude of those who invented the first constellations (by looking at the size of the cap around the south pole that has no ancient constellation figures), and even to estimate when they lived (by noting that the centre of that cap has moved because of precession). The conclusions are that the constellation makers lived at around 35°–36° north (south of Greece but north of Egypt), at some period between 3000 BC and 2000 BC; the most probable candidates seem to be the Babylonians and Sumerians. However, the modern set of constellations really starts with the 48 listed by Ptolemy in the *Almagest* of around 150 AD. Ridpath then traces the development through the Arabic influences and the later (post-1500) additions of extra constellations, particularly by Dutch, Polish, and French explorers and astronomers, until the final definitive list of 88 agreed by the IAU at its first General Assembly in 1922. The precise boundaries came later, in 1928, resulting from the work of the Belgian astronomer Delporte.

In Chapter 2, Ridpath turns to the related topic of star maps, or atlases, starting with the oldest known celestial globe, called the Farnese Atlas, because it is a 2nd-Century-AD sculpture of Atlas carrying the globe, which shows constellation figures but no individual stars. Although celestial globes of greater accuracy did exist in Europe in the 2nd Millennium (the RAS owns a pair of celestial and terrestrial globes), most later representations of the sky were on flat surfaces, initially on astrolabes and later on paper; the earliest surviving paper map is Chinese, dating from the mid-7th Century AD, but the Chinese constellation figures were very different — smaller and more numerous, with names relating to various facets of daily life — and were unknown in the West until the 17th Century. The Arabs produced depictions of the Ptolemaic constellations, but real progress came in 1515 with the production of the first printed star chart using a pair of woodcuts by Dürer, one for each hemisphere. The next landmark, based mainly on Tycho Brahe's detailed observations, was Bayer's *Uranometria* of 1603, the first major printed star atlas, depicting some 2000 stars, followed in 1690 by Hevelius's catalogue and atlas, the last to be based entirely on naked-eye observations, many by Hevelius himself. The first major atlas based on telescopic observations was produced by Flamsteed, using observations made at the new Royal Observatory but not published until 1725, after his death, with the atlas appearing in 1729, still with constellation figures superimposed on the stars. The greatest of these old-style atlases was Bode's *Uranographia* of 1801, which was the first to depict virtually all the naked-eye stars, plus some fainter ones; it contained over 17000 stars. After that there was a gradual transition to more scientific mapping, without the distraction of the constellation figures. An example of this evolution is the 1843 atlas of Argelander; it still has constellation figures, but they are printed in red and are much less obtrusive.

Chapter 3 is the main text, 148 pages containing in alphabetical order an entry for each of the 88 IAU constellations, from Andromeda to Vulpecula, plus one for the Milky Way. Each constellation entry starts with a small table listing the genitive of the name, its abbreviation, the size ranking, the origin and (if it exists) the Greek name, which is sometimes quite different from the familiar Latin one. The smallest is the Southern Cross, and the largest is Hydra, which stretches over nearly a quarter of the sky. The text in each entry describes the myth (if any) behind the figure and also gives some information about the content of the constellation. Each entry is accompanied by an illustration of it from one of the classical atlases. The length of the entry can be as short as a page or as long as five pages, partly depending on how much mythology is behind the name. Of course, the modern constellation names tend not to have mythological origins — for example, the 62nd-largest constellation, Antlia (the air pump), named *La Machine Pneumatique* by Lacaille on his chart of southern stars in 1756, but Latinized on his 1763 second edition and shortened by Francis Baily in his 1845 catalogue. Other examples include Apus, Caelum, Circinus, Fornax, Lacerta, Mensa (named by Lacaille after Table Mountain), Octans, Reticulum, Scutum (originally named for political reasons), and Volans. These, like the other modern examples, are mostly southern constellations and many have interesting notes about why they were named. The Milky Way is included in order to give a small sample of the many stories about it, and its different names in different cultures.

Because constellation figures are human inventions, it is not surprising that some proposed names and patterns were rejected along the way, and in his final chapter Ridpath gives an interesting account of a set of two dozen constellations that achieved acceptance for a period and are found on old maps (almost every entry is accompanied by the appropriate map) but have not made it into the IAU-approved list, at least in their original form (for example, the original Argo Navis of Ptolemy was subdivided in the 18th Century into the current Carina, Puppis, and Vela). The story behind the naming is given in each case.

I thoroughly enjoyed reading this beautifully-illustrated book, and it will be a lovely reference book as well. It is meticulously researched and very well written, and I can recommend it to anyone interested in the constellations from a historical or mythological perspective. — ROBERT CONNON SMITH.

A Most Improbable Journey: A Big History of Our Planet and Ourselves, by Walter Alvarez (W. W. Norton & Company), 2017. Pp. 246, 21 × 14 cm. Price $15·95 (about £12) (paperback; ISBN 978 0 393 35519 2).

Improbable Journey is actually a fascinating volume, particularly the four geology chapters, not surprising, since that is author Walter Alvarez's territory. He is best known to most of us as part of the team that found an iridium-rich layer at the boundary between Cretaceous and Tertiary (now called Paleogene) formations in Gubbio, Italy, providing evidence for an asteroid-ish impact as a major contributor to the floral and faunal extinctions then. He makes a case that the scientific revolution can be said to have started with the Portuguese voyages of exploration before 1500 and thus that geography/geology was the first science. Copernicus and astronomy are the competitors he has in mind, though Wootton (reviewed in these pages **138**, 314, 2018) proposed the 1572 Tycho *nova stella* (but also astronomy).

The astronomy and cosmology chapter is a good deal less satisfactory. Readers are informed that "cosmic inflation had taken a nearly uniform early universe and magnified its tiny initial fluctuations in density into a universe with major density contrasts". Later, "the explosive jamming together of hydrogen nuclei produces all the elements heavier than iron, and the explosion itself scatters every element throughout the region of space surrounding the supernova, where they can be incorporated in new stars". The first half of that quote appears for its scientific mis-statement, the second half for its grammatical infelicity.

Alvarez thinks very highly of Martin Rees's *Just Six Numbers* (Basic Books, 2003) and carries away the message that "the human situation is balanced on a knife edge of improbability, which again is the principal theme of this book". The readers who have the best chance to ask Rees whether this is what he meant are those who happen to be perusing those words in an airplane over the Atlantic Ocean, whose gradual widening is one of many geological events that, Alvarez tells us, have played a role in determining biological and human evolutionary events.

The 'contingency' theme waxes stronger as the author moves from the Universe to the Earth to life, pointing out that the Chicxulub impact meets all the criteria for contingency, having been rare, unpredictable, and significant. The pioneering paper is the first reference noted for the first chapter (L. W. Alvarez, W. Alvarez, F. Asaro & H. V. Michel, 'Extraterrestrial cause for the Cretaceous–Tertiary extinction: Experimental results and theoretical interpretation' in *Science*, **108**, 1095, 1980). And now a 'most improbable' factoid: this is the only mention of Luis Alvarez (physics Nobel 1968 and mapper of pyramids with cosmic-ray-muon secondaries) in the book. Great grandfather Luis Fernandez Alvarez gets the better of two pages and a photo for the contingent action of emigrating from Spain to Cuba at age 7 (1860) after the death of his father, and becoming a doctor in Hawaii and California. Completely missing without even a footnote is medical doctor and grandfather Walter C. Alvarez, for whom presumably the author was named. His syndicated column of medical advice and information appeared in the *Los Angeles Times* in the 1950s, and I still remember its common-sense tone.

The visuals are largely limited to serviceable maps, some drawn by the author, and black-and-white photos of astronomical and geological entities, most with inadequate angular resolution and contrast. The acknowledgements include wife Milly Alvarez, and Don, Jean, and Helen, of that surname, along with dozens of others from various disciplines and nations. Lisa Randall is the only obvious cosmologist, though there is a cover blurb from Sean Carroll.

The author agrees with those who proposed to the IAU in 2018 August that the Hubble law should be renamed. But his candidate is Hubble–Humason, not Hubble–Lemaître.

Conflict of interest statement: My copy of *Improbable Journey* was a gift from someone at W.W. Norton, after I complained about being offered examination copies of astronomy textbooks, none of which are ever anywhere near as much fun as *Just Six Numbers* or the volume to hand. — VIRGINIA TRIMBLE.

Juan Valderrama y Aguilar, pionero de la astronomia canaria (1869–1912) [Juan Valderrama y Aguilar, pioneer of Canarian astronomy (1869–1912)], by M. Vázquez Abeledo & J. Sánchez Almeida (Organismo Autónomo de Museos y Centros, Instituto Astrofísico de Canarias, Tenerife), 2018. Pp. 116, 24·0 × 17·0 cm. Price €10 (about £9) (paperback; ISBN 978 84 88594 87 7).

In some cases, a brief biography of an unknown common citizen occasionally surprises the scientific community by showing that astronomy can be not only a science but also a hobby. Without any doubt, this is one of those cases. Several years ago, a lady left a set of old handwritten notebooks in the library of the Astrophysics Institute in the Canary Islands (IAC). No document was presented showing the notebooks to be a voluntary donation, although the library has kept them well preserved to date. José Antonio Bonet was the first astronomer of the IAC who noticed those manuscripts and their significance. Much later, a group of astrophysicists found in those notebooks the third observation in history recorded about the white-light flare in 1886 (Vaquero *et al.*, 2017, *Solar Physics* **292**, 33). Unfortunately, the author of those manuscripts remained a mystery.

But recently Manuel Vázquez and Jorge Almeida have managed to shed some light on the biography of that mysterious amateur astronomer who scanned the sky with his little refractor telescope in the late 19th Century, showing that he kept up to date on astronomical news thanks to specialized magazines. His name was Juan Valderrama y Aguilar. He belonged to a well-regarded family on the Islands. His granduncle, Francisco de Aguilar y Fuentes, was also an amateur scientist who helped the first professional scientists to confirm the magnificent conditions of the island of Tenerife as a centre for scientific development, particularly in the field of astronomy.

This book provides a thorough study of Valderrama's life and works. His social, educational, and family environment is studied in detail. There is a remarkable connection between the astronomical studies of this character and the development of official meteorology in the Canary Islands at that time. In fact, Valderrama was the director of the Meteorological Observatory of Santa Cruz de Tenerife from 1903 June to 1912 March, and there is also evidence that a relative of his kept regular meteorological records in a small private station. The scientific results together with Valderrama's notebooks are detailed in this book, including the white-light flare in 1886, sunspot observations, observations of the Moon, conjunctions of stars and planets, the light curve of Nova Persei, and observations of Mars, among others. The book also includes a series of open questions around his life and a detailed glossary, as well.

In conclusion, this book should be a must for those readers who are interested in the history of astronomy of the Canary Islands before the arrival of the era of large installations and telescopes in the second part of the 20th Century. — JOSÉ M. VAQUERO.

On the Future: Prospects for Humanity, by Martin Rees (Princeton University Press), 2018. Pp. 256, 19·5 × 13 cm. Price £14·99 (hardbound; ISBN 978 0 691 18044 1).

Many astronomers are futurologists at heart. We study things that are changing with time, and thus we are interested in endpoints. So it is not surprising that our Astronomer Royal, the author of the book under review, gets out his metaphorical crystal ball and tries to predict the future.

Rees wears three hats: those of a scientist, a citizen, and a concerned human being. Many medieval people were miserable but there was little that the contemporary power-broker could do about it. The miserable of today are different. The global communication network now ensures that they know what they are missing; and today's science and technology offers huge opportunities. We are living in the first historical era in which humanity can critically affect our planet's entire habitat, climate, biosphere, and the supply of natural resources. Many folk worry too much about the positive gradient of global warming and too little about the negative gradient of global elbow room. But Rees is a techno optimist. He stresses the fact that, when it comes to the evolution of life and intelligence, today's humans are not the end-point; in fact we might not even be the half-way point. Our Solar System is barely middle aged, and if we avoid self-destruction a post-human machine age of artificial intelligence beckons.

Astronomically Rees is fascinated by the prospects of their being alien life forms on some of the multitude of planets we have recently discovered. He underlines the fact that the only shared culture will be our mathematics, physics, chemistry, and astronomy. Much is made of the Big Bang and the possibility there were many of them resulting in a multitude of universes. I was rather intrigued by the fact that the book assumed that the physics and astronomy we know today is all correct. This I found rather odd because it has never been like that in the past. Also there is the possibility that science may soon 'hit the buffers'. There is the strong likelihood that there are truths about Nature that are far too complex for us humans to grasp.

This book is extremely well written, thought-provoking, and stimulating. I recommend it unreservedly. It is abundantly clear that when it comes to 'wise men' Martin Rees is very close to the front of the queue. Space-ship Earth is hurtling through the void and many of its passengers are anxious and fractious. Our future depends on us making wise decisions, and the 'us' is not just the scientist, but all the citizens of the planet. Rees's glass is half full — he is a cautious optimist. The end of the world does not have to be nigh, and this book is a great encouragement for us all to pull together for better times. — David W. Hughes.

Galileo Unbound: A Path Across Life, the Universe and Everything, by David D. Nolte (Oxford University Press), 2018. Pp. 335, 22·5 × 14·5 cm. Price £25 (hardbound; ISBN 978 0 19 880584 7).

Author David Nolte has given us a history of large parts of science, including quite a lot of astronomy and cosmology, focussed in a unique fashion. After a glance at 'the Greeks', he begins with parabolic trajectories as understood by Galileo and ends (briefly) with a multiverse, though the last main chapter deals with non-linear oscillations in (mostly) living systems.

I will attempt to describe the 'Ariadne's thread' between, without quoting so much as to commit deliberate plagiarism. In the phrases that follow, "it" means the curve of a parabola, which "Newton held aloft … Maupertuis used providentially … Lagrange freed from three-space … Riemann warped … Boltzmann averaged … Poincaré saved … Einstein refracted … Heisenberg tried to abolish … Feynman went crazy over … Arnold turned into the Cheshire cat … Lorenz released … Darwin grew into a tree … Rayleigh and van der Pol condensed for networks of synchronization … and we hold … hoping it will give us a glimpse into our futures."

We also meet Misner, Thorne & Wheeler (incorrectly credited with the invention of the phrase "black hole"), Stirling Colgate, Bethe, Bohr, and Brahe, and many other old friends.

My favourite 'clock striking thirteen' moment happens on p. 226, "In 1976, computers were not common research tools, although hand-held calculators now were". Even the inept Trimble was using computers for astronomy from 1966 at Caltech and from 1968 in Cambridge (UK), but the Hewlett-Packard HP-23 was very new in 1972.

There are other misspeaks: for instance, Karl Schwarzschild is said to have died on the Russian front (actually he was invalided home with pemphigus, rare now, but not then).

But I would like to nominate for the "Hey, whose side are you on?" award of the month to Nolte's description (as part of a paragraph on Karl Weierstrasse) of the outcome of the 1815 (post-Napoleonic) Congress of Vienna: "Among of the consequences [the "of" doesn't belong] of the Congress was a consolidation of Prussian power and influence, which had the fortunate side effect that it established a stable environment across much of Germany that lasted over a century, fostering the intellectual rise of German science and mathematics in the nineteenth century." Of course that "stable environment" lasting more than a century takes us somewhere to the middle of World War I, another product of Prussian power, and not, in my view, "fortunate".

Not wanting to end on a dismal note, however, I take you to p. 113, where we are told that Georg Cantor "sat on lectures by Weierstrasse and Kronecker." Owing, we suppose, to a shortage of chairs in his office. — VIRGINIA TRIMBLE.

Mercury, by William Sheehan (Reaktion), 2018. Pp. 184, 23 × 18 cm. Price £25 (hardbound; ISBN 978 1 78914 012 5).

Mercury may be a rather small planet — not much bigger than our Moon and smaller than Ganymede — but it has captured our attention since ancient times and still fascinates us to the point where we are prepared to part with a small fortune to send the ESA-led *BepiColombo* spacecraft to take a close look. Since it will not arrive at its destination until 2025, it leaves us plenty of time to 'bone up' on the subject, and the present book by William Sheehan is the perfect way to do it. At a level accessible to upper-school students, this well-written account could inspire scientifically-inclined youngsters to think about a career in astronomy knowing that when they have left school, passed their undergraduate studies, and done a PhD, a whole lot of new data on Mercury will be flowing in ready for them to analyse.

In a short Chapter 1, Sheehan outlines the difficulties of observing an object that never strays far from the glare of the Sun, while Chapter 2 discusses the motion of Mercury, starting with the varied pre-Copernican models but ending up with today's orbital parameters; both chapters — and those that follow — are suitably embellished with diagrams, historical plates, and superb colour images.

Chapter 3 considers what telescopic observers have seen across the years, including during transits, which leads into the long Chapter 4 where claimed observations of surface features led to estimates (guesses?) of Mercury's rotation rate, for a long time thought to be 88 days; it's a question that was only settled relatively recently by radio astronomers at 59 days. It might have been better in that chapter to have rather larger reproductions of some pages from Schiaparelli's logbooks; even a magnifying glass did not help me very much to see what was going on.

Progressing into the modern era, we get a good description of Mercury's surface, primarily through the observations by *Mariner 10* and *MESSENGER*. The pictures in Chapter 5, some in colour, are quite delightful and whet the appetite for future revelations by *BepiColombo*.

Chapter 6 takes us on an historical hunt for Vulcan, once thought to be a planet orbiting the Sun even closer than Mercury and possibly responsible for the strange advance of the latter's perihelion. But it was never found and it remained for Einstein's theory of relativity to explain all.

This beautifully produced and modestly inexpensive book concludes with a glossary, an appendix of basic data on the planet, a list of craters, a set of end-notes, and full index. All in all, a splendid primer for those future astronomers. — DAVID STICKLAND.

Astrophysics of Red Supergiants, by Emily M. Levesque (Institute of Physics Publishing), 2018. Pp. 88, 26 × 18·5 cm. Price £99/$150 (hardbound; ISBN 978 0 7503 1330 8).

The volume under review is the first of a new series of ebooks established by the American Astronomical Society and published by the Institute of Physics in London. The principal goal of the series is "to create an exciting new collection of astronomy and astrophysics ebooks" to support the AAS mission to further "humanity's scientific understanding of the universe". These books in hard copy and electronic form will range from "short introductory texts on fast-moving areas, graduate and upper-level undergraduate textbooks, research monographs and practical handbooks". The IOP webpage shows that four ebooks have been published and a further 27 have been commissioned with publication expected before 2020 November.

Astrophysics of Red Supergiants, by Emily M. Levesque, published in 2017 December is a research monograph on these evolved stars with masses from about 8 to 40 $M_{\odot}$. Brief chapters review recent literature on aspects of red supergiants from their interior through their envelopes and atmospheres to their stellar winds and dusty circumstellar shells. Much is assumed of the reader. For example, Chapter 4 is devoted to mass loss and dust production where published mass-loss rates are compared beginning with the venerable Reimers law, but there are no descriptions of how a mass-loss rate and its uncertainty are estimated from observations of gas and/or dust in a star's wind and dusty circumstellar shell. Readers with an observational bent will enjoy the chapter on red supergiants and supernovae Type II-P which highlights SN 2008bk and SN 2005cs, where imaging before and after the explosion clearly shows the red supergiant which exploded. The final chapter foretells the future of observational work on red supergiants including the likely contributions of the *LSST* and the *ELT*s.

Graduate students with interests in massive stars and aspects of astrophysics affected by these luminous stars would benefit by reading this first volume in the AAS–IOP series of ebooks. Unfortunately, the cost is surely beyond the pocket of a student and others too. Perhaps, the student's university library will subscribe to the series and provide students with an accessible electronic version. — DAVID L. LAMBERT.

Understanding Stellar Evolution, by Henny J. G. L. M. Lamers & Emily M. Levesque (Institute of Physics Publishing), 2018. Pp. 322, 26 × 18·5 cm. Price £99/$150 (hardbound; ISBN 978 0 7503 1279 0).

This book is based on notes for a lecture course delivered by the authors for more than a decade to students at the University of Washington. That origin leaves a clear signature in the choice and level of content (suitable for undergrad courses at 2nd-year and up, or early-stage postgrads); in the well-organized structure; and in the format and the ancillary materials. Although both authors are stellar astrophysicists, neither principally researches in stellar evolution *per se*. That's no handicap at this level, and if anything their stated aim of providing a feeling for, and physical understanding of, the central concepts of stellar structure and evolution (as opposed to rigorous mathematical derivations) is well served by this modest distance.

Broadly speaking, it's a book of two halves. The first covers the essential physical foundations — fundamental structure equations, opacities, energy transport, and fusion, leading up to polytropic models and star formation. The second reviews stellar evolution as we believe it actually happens, largely divided into sequential stages of nuclear burning, but with substantial asides into topics such as 'pulsating stars', 'rotation and stellar evolution', and 'close binary evolution'.

Each of the 30 chapters reads like the basis for a roughly one-hour lecture (the course on which the book is based was delivered as twenty 80-minute lectures). Questions intended to be posed during delivery are embedded in the text, and homework exercises are also provided (answers available only from the authors!). Each chapter concludes with a set of useful references to textbooks, review papers, and important primary sources. Associated Powerpoint/Keynote slides can be obtained on request from the publishers; these aren't as slick and polished as the book itself, but would make a useful starting point for lecturers developing a course from scratch.

While I was glad to have a (really rather nicely produced) hardcopy edition, even this is branded as an "IoP ebook", and pdf, ePub, and kindle editions are all available (at only a very slight discount compared to the hardback). I suppose this might explain the conspicuous absence of an index (given that the e-versions are directly searchable), but that will still be a distinct disadvantage to anyone limited to the print version, a reasonably detailed contents listing notwithstanding. However, you may well find, as I did, that you have 'free' access (*i.e.*, pre-paid by someone else) to one of the machine-readable editions, through an institutional library arrangement.

One tiny quibble: from a didactic perspective, I think it's a shame that the book uses cgs throughout (the "we've always done it this way, so we must continue to do it this way" argument is wearing a bit thin with me), including in the appendix of astronominical constants, where the reference values for solar parameters aren't IAU-recommended nominal values. Otherwise, I unreservedly enjoyed browsing this lucid text, and will certainly co-opt elements for my own lectures. — IAN D. HOWARTH.

Accessory to War: The Unspoken Alliance between Astrophysics and the Military, by Neil deGrasse Tyson & Avis Lang (W. W. Norton), 2018. Pp. 576, 23·5 × 15 cm. Price $30 (about £23) (hardbound; ISBN 978 0 393 06444 5).

Neil deG. Tyson, protagonist and sometimes hero of very many initiatives in education and public outreach, is best known between hard covers as the author of *Astrophysics for People in a Hurry*, which has spent more weeks on the *New York Times* best-seller list than the lifetime of most laboratory rats. *Accessory* is aimed at people in a bit less of a hurry and is dedicated "to everybody who has ever wondered why astrophysicists have jobs at all".

You might read the present volume either as a history of astronomy from the Sumerians to the *Spitzer Space Telescope*, with a good deal of war thrown in, or as a military history from Sun Tzu to the Strategic Support Force of the People's Liberation Army with a good deal of astronomy thrown in. I had expected at least a short 'dialogue' with Martin Harwit's 2013 *In Search of the True Universe* (reviewed in these pages by Phillip Helbig: **134**, 217, 2014), but in fact only one of his earlier books is referenced. Harwit's view has been that the development of astronomy has been so heavily driven by what military research has provided that we may be missing important phenomena (though he gives no examples).

Tyson and Lang, in contrast, treat the relationship between astrophysics and the military as a more equal partnership, and I am certainly in no position to take sides between them (though my father and husband both held high-level security clearances at various times, nobody in his right mind would tell me anything he didn't want blabbed to the four major winds and the 'Merry Little Breezes').

A few astronomical misspeaks shortly, but first a fascinating direct quote that bears on the question of "who invented 'tired light' as an alternative to cosmological redshift?" The passage dates from 1878–79, comes from Thomas Alva Edison, and concerns the use of his tasimeter, an infrared detector whose unsuccessful application to solar eclipses is mentioned in S. Cottam & W. Orchiston's 2015 *Eclipses, Transits, and Comets in the Nineteenth Century* (reviewed here by Steve Bell: **135**, 307, 2015). He wrote, "But now there comes a promise of an extension of positive knowledge to fields of space so remote that light is tired out and lost before it can traverse the intervening distance." He also mentions its possible use for detecting nearby burnt-out stars and feebly-reflecting planets, which indeed modern infrared astronomy has given us. Such tired light preceded Zwicky's by a bit more than 50 years, but counting it as a prediction of redshift into the IR would be unreasonable.

Other items are less happy. Here are two examples: (*i*) from page 213–214, "A scintillator is a tiny block of energy-sensitive material (caesium iodide, for instance) that pumps out tiny flashes of light — charged particles — each time a gamma-ray barrels through it." The next paragraph clarifies that cosmic rays are charged particles, leaving the gamma rays to be the light, I suppose. And (*ii*) from p. 131, "A year later, Curtis would be on the wrong horse in a highly publicized debate about whether the Milky Way was the entire universe or whether the spiral fuzzy objects seen dotting the night sky were other galaxies, rendering the actual universe vastly larger than previously imagined." In fact, Heber Doust Curtis maintained that the 'faint fuzzies' were other galaxies (though his distance scale was too small), while it was Harlow Shapley who claimed the Milky Way was the only game in town (and his distance scale was too big). So Curtis was right about the point that Tyson & Lang raise, though he was wrong about the official title of the discussion, 'The Distance Scale of the Universe'.

Please provide your own assonant witticism about gems and glass, because the very same page goes on to discuss which European companies were producing the best optical glass and when. Chance Brothers (Birmingham) and Parra Montois *et Cie* (Paris) were new to me; Carl Zeiss and Ernst Abbe (Jena) are old friends, but there with them was a newly-PhD'd chemist Otto Schott (whose dissertation topic was the fabrication of glass), the eponym of Schott Glass Company.

If you are very lucky, you might someday soon find yourself at an astronomy conference where the exhibit hall includes a stand attended by a modern representative of the company, who might possibly offer you a small, pink Pyrex container labeled 'Schott glass', just right to be a shot glass for whisky. I happily here raise mine in toast to Tyson & Lang, whose jewels really do outweigh their gewgaws.

Conflict of interest statement: my copy of *Accessory* was not originally intended as a review copy, but was purchased for full price at an airport gift shop. — VIRGINIA TRIMBLE.

High Time-Resolution Astrophysics: Canary Islands Winter School of Astrophysics, Volume XXVII, edited by Tariq Shahbaz, Jorge Casares Velázquez & Teodoro Muñoz (Cambridge University Press), 2018. Pp. 194, 25 × 18 cm. Price £110/$140 (hardbound; ISBN 978 1 107 18109 0).

In the series of Canary Islands Winter Schools of Astrophysics, the one which took place in late 2015 was devoted to high-time-resolution astrophysics (HTRA). The book's back-cover blurb mentions how HTRA techniques, driven strongly by areas such as X-ray and pulsar astronomy (whose needs for high time-resolution were particularly acute), are now being adopted across the electromagnetic spectrum, implying applications to a broad range of areas. It goes on to state that "this volume gives a practical overview of, and introduction to, the tools and techniques of HTRA". Contrary to those claims, however, is the fact that five of its six chapters are primarily concerned with X-ray sources, white dwarfs, neutron stars, and black holes. Important and necessary though they may be, it leaves almost untouched the plethora of other instances of rapid variability that astronomy exhibits: stellar-atmospheric turbulence, supernova or nova outbursts, solar flares, ionospheric disturbances, spasmodic mass loss from stars, or shocks related to massive outflows (to name only a few). To give the book's authors due credit, however, it should be mentioned that the 'Introduction to Instrumentation' for HTRA (by P. A. Charles) gives a very readable survey of the history of HTRA and the key developments that seeded changes, while the final chapter (by E. C. Ferrara) goes some way to redress the astrophysical balance by focussing on gamma-ray data. There were actually seven lecturers at this Winter School but only six have contributed chapters; the topic of the seventh was not disclosed. All but the two instrumentation chapters contain substantial amounts of theory.

The book includes two chapters on instrumentation for HTRA, one each on radio observations and theory of pulsars and X-ray binaries, the physics of compact objects (effectively X-ray and neutron-star binaries), X-ray emission from black-hole and neutron-star binaries, and a final chapter on incorporating gamma-ray data into HTRA. While there are substantial discussions on causes of rapid time variability and how to interpret large swathes of observations in that context, the chapter on gamma-ray data presents instead the different challenge of extracting science from gamma-ray events when the time signatures of an object are significantly shorter than the intervals between individual detections, and it teaches the maximum-likelihood-analysis methods that are needed in those circumstances.

Each lecturer delivered four lectures, though one does not know how much introductory material was excluded for the purposes of this publication. The first chapter gives a fairly broad view of the relevant developments in theory. Of 135 citations in its bibliography fully one-third are pre-2000, so it offers enough of the subject's past to educate its readers without sending them to the ADS to try to piece together where it all came from. The chapter by Charles on instrumentation defines efficiently the growth and development of interest, understanding, and expertise in HTRA (primarily at X-ray wavelengths) by realizing technological advances. On the other hand, the second chapter on instrumentation is less useful since it is mostly given over to the high-speed photometer *ULTRACAM*. It is also riddled with acronyms that can baffle the uninitiated; fortunately the book offers a glossary of acronyms (it runs to three pages).

I criticize the Editors for not removing redundant comments like "based on a series of lectures ...". We know that. It is also a pity when none of the editors is a native English speaker, and phrases like "allows to [determine]" slip in unchecked. But all in all, the book is a valuable source of information and current ideas and is to be recommended as a class text. Its balance could have been improved by slotting the chapter on gamma-ray data in the middle rather than leaving it to the end as an after-thought (its author was also the only woman lecturer). — ELIZABETH GRIFFIN.

Workshop on Astrophysical Opacities (ASP Conference Series, Vol. 515), edited by Claudio Mandoza, Sylvaine Turck-Chièze & James Colgan (Astronomical Society of the Pacific, San Francisco), 2018. Pp. 324, 23·5 × 15·5 cm. Price $88 (about £69) (hardbound; ISBN 978 1 58381 914 2).

The proceedings of a workshop held in Michigan in 2017 August, this volume presents a lively and informative cross-section of current work in one of the most fundamental areas in stellar and planetary astrophysics. Just how important is indicated by a memorial page which sadly records the relatively recent passing of six colossi of atomic astrophysics. On a more encouraging note, a quite extensive use of colour and well-edited copy reflect the lively and interesting science discussed. Various sections address the underlying science drivers, progress on the computation (mostly) and measurement (uniquely) of opacities, applications to astrophysical processes and environments, and the generation of new atomic data. Highlights include an update on experimental measurements of iron opacities under stellar conditions, and evidence that the modelling of atomic diffusion inside stars is becoming mainstream. The volume provides a refreshing overview of work in many laboratories, and includes reviews of historical progress, pedagogical descriptions of contemporary codes, and candid assessments of current data. The forward look, written in 'White Paper' format, is particularly useful. This was a meeting the reviewer would dearly have liked to attend, given time and resource. Having a copy of the proceedings goes part way to make up the deficit, but he will have to find another way to add his wish-list of atomic data for exotic ions. — C. SIMON JEFFERY.

OBITUARY

George Alan Wilkins (1929–2018)

I am sorry to report the death on 2018 December 16 of George Wilkins at the age of 89, following a period of failing health. George's career spanned almost the entire period of the Royal Greenwich Observatory (RGO) at Herstmonceux where, as anyone who had dealings with him will testify, he was a meticulous record keeper and tireless advocate of the importance of archives. Access to his personal accounts has (uniquely for me) provided a wealth of biographical material and greatly eased the compilation of this appreciation.

George's parents were from the Black Country in the West Midlands and moved to Croydon in 1928, just before George was born. He attended West Thornton Elementary School before winning a scholarship in 1939, aged 10, to Whitgift Middle School, one of the local grammar schools. Here his interests in mathematics and physics came to the fore, and a school prize awarded in 1943 was used to buy R. H. Baker's *Introduction to Astronomy*. His sixth-form studies for the Higher School Certificate resulted in distinctions in physics and applied mathematics, plus the decision to stay on for a third year to prepare for university scholarships. Having been offered places at Queen Mary College, London, Selwyn College, Cambridge, and Imperial College, London, George took up a Royal Scholarship to read physics at Imperial. In 1948 he gained an upper-2nd-class degree in physics and opted to take a further year of mathematics for which he achieved a 1st-class degree in 1949. The award of a two-year research grant enabled him to enter on a PhD under Professor A. T. Price, analysing daily variations in the Earth's magnetic field. This involved much routine numerical work, performed with the aid of an electromechanical calculator, and the development of new techniques for interpolation and analysis. Early in 1951 he applied for, and was accepted, for a post at HM Nautical Almanac Office (HMNAO) where he could continue his PhD after the expiration of his grant.

George joined HMNAO as a scientific officer in 1951 October, early in the decade which saw the transfer of the Royal Observatory from Greenwich and the consolidation on the Herstmonceux site of sections scattered by the war: the NAO from Bath, the Time Department from Abinger, and the Chronometer Workshop from Bradford-on-Avon. In the first few weeks, like all new recruits to the NAO, George undertook the basic training in numerical calculation (using a Brunsviga manually-operated calculating machine) and proof-reading, with particular emphasis on accuracy, efficiency, and mental arithmetic. Together with other early recruits he was accommodated in the on-site hostel (one of the temporary buildings south of the castle) and was able to devote his spare time to the completion of his thesis which was submitted after a few weeks. So began a career, ideally suited to his talents, which was to see him immerse himself in the then slightly unfashionable disciplines of positional astronomy and navigation, travel the world in pursuit of excellence and unification in those fields, and rise to become Superintendent of HMNAO in 1970, a post he held until his retirement.

The value of the office's publications, and indeed its reputation, depended critically on every tabulated quantity therein being correct, and George soon buckled down to the routine which was, like all HMNAO staff, to occupy the first two hours of nearly every working day for the next 20 years: proof-reading. Among his first projects was planning the transfer of the huge amount of labour in calculating the ephemeris of the Moon — processes which involved summing hundreds of terms based on E. W. Brown's lunar theory — to exploit the power of punched-card machines. Later, after the NAO had acquired its first electronic computer, he developed a Fortran program to carry out that task, which was to form the international standard for many years, until supplanted by numerical integrations.

In 1957 February George and his family set sail for the USA on the *SS Queen Elizabeth*, bound for Washington where he was to spend a year at the US Naval Observatory as part of an exchange between the NAOs of the two countries. Rather than work to do directly with almanacs he was assigned the task of computing improved orbits for the satellites of Mars, particularly Phobos which was thought to have an anomalous secular acceleration. This was to be a continuing interest and he collected all published positional observations of the satellites and, many years later, he and Andrew Sinclair used them to make a complete analysis. That visit also served to strengthen the very fruitful relationships between the two offices which had been nurtured by Donald Sadler, then Superintendent of HMNAO. This close collaboration manifested itself in many ways and continues to this day. On his return George was responsible for editing the first edition of the *Explanatory Supplement to the Astronomical Ephemeris and to the American Ephemeris and Nautical Almanac*, published in 1961, a volume that set out in detail how every tabulated quantity was derived. It, and the further editions edited by Dr P. K. Seidelmann of the USNO, have proved invaluable references for anyone interested in positional astronomy and Solar System studies.

At about this time the RGO recognized the need to enhance its computing capability and George was at the forefront of the moves to acquire an electronic computer. His hope of an IBM machine for reasons of compatibility with the USNO was not to be, and the tendering process and eventual delivery took a long time. But the use of a series of ICT machines over the next few years greatly eased the load on human computers, as well as providing a service for other Observatory departments. One aspect which George fully embraced was the opportunity they presented of producing the final printed copy for publication directly from computer output, using a high-quality card-controlled typewriter, rather than having the pages typeset by a printer with all the attendant risks of introducing errors.

By this stage he was increasingly involved on the international scene, especially through his membership, and later 3-year presidency (1967–1970), of the International Astronomical Union's Commission 4 on Ephemerides. He was a leader in the development of the 1976 IAU System of Astronomical Constants and this was introduced into the 1984 edition of *The Astronomical Almanac* which had been a joint publication of the two almanac offices since 1981. Other innovations at that time included adoption of a new reference system and the use of numerical integration in the preparation of ephemerides. As mentioned above, one of George's enduring concerns was the need for genuinely global standards and nomenclature in the astronomical world in order to avoid confusion and duplication of effort. In 1989 he compiled the IAU's official *Style Guide* aimed at encouraging clarity in all astronomical publications, uniformity in the naming of astronomical objects, use of appropriate units, consistency in the style of references and abbreviations, and recommendations for the use of text-processing software. He undertook a revision of the *Guide* in 1995.

The first laser retro-reflectors were placed on the Moon by Apollo astronauts. Some time later George was approached by the laser group at Hull University about the possibility of ranging to them from the UK. He asked Andrew Sinclair to review the nascent lunar laser ranger in Australia and they concluded that ranging to the Moon was both difficult and expensive. Meanwhile, the launch in 1975/76 of the geodetic satellites *Starlette* and *Lageos* had widened the variety of potential applications relevant to UK interests and at lower cost. George was then a leading influence in all the discussions and plans which led to SRC funding the UK Satellite Laser Ranging station installed on the Herstmonceux site where it thrives to this day thanks to the fact that George was successful in ensuring that its work was not compromised, or even abandoned, by proposals to relocate the station to Cambridge. George's primary interest lay in the promise of the new technique being used to replace the *Photographic Zenith Tube*, also on the Herstmonceux site, as a more accurate way of determining Earth-rotation parameters. This expanded into a much wider international project to evaluate the contributions of various techniques such as laser ranging and radio interferometry to the determination of universal time and polar motion — project MERIT (Monitoring Earth Rotation and Intercomparison of Techniques) — which George chaired and was very successful in improving the accuracy of Earth-rotation parameters. Project MERIT led directly to the setting up of the International Earth Rotation and Reference Systems Service (again chaired by George for its first two meetings) which is the body responsible for maintaining global time and reference-frame standards. He continued to work on this and other Commission 4 matters for more than a decade after his formal retirement and was also a regular visitor to the RGO archives, by then housed in the Cambridge University Library, while engaged in the writing of his personal history.

In the 1970s George became responsible for the oversight of the library and archives. A lack of resources and of interest had led to neglect of the latter and this prompted the appointment of Janet Dudley as the Observatory's first professionally qualified librarian and archivist. There followed a decade in which the organization of the library was rationalized: HMNAO library was integrated with the main library in the castle; full use was made of Universal Decimal Classification system; and particular attention was paid to the Airy Collection of rare books and proper safeguarding of the archives. Hitherto the archives of three centuries had been poorly treated and haphazardly stored in a deep cellar in the West Building. George and Janet made the case to the Public Record Office that the RGO should retain its own archives and drew attention to need to provide extra facilities and staff for their processing and conservation. This bore fruit in the establishment of a project to compile a comprehensive schedule of the contents of the archives and the setting up of a conservation laboratory in the castle to prolong the life of vulnerable records. Use of the archives by visiting researchers increased during this period. When the observatory moved to Cambridge in 1990 the archives were transferred to the University Library under the care of Adam Perkins.

For one with such a tidy mind and tidy habits (at the end of each day everything on his desk disappeared into the drawers) his inimitable handwriting proved challenging to all but a few. A stickler for accuracy, he expected high standards from his staff who were kept informed of what was going on in circulated notices. Any written work submitted for approval was almost certain to come back covered in red ink, a practice which some found daunting at first but came to appreciate as a very good education in how to express oneself accurately and succinctly. He was very effective on committees, as delegate or chairman, giving everyone a fair say and not pressing his own views to excess (except in one instance: his dogged insistence, ultimately unsuccessful, that the name Greenwich should be retained somewhere in the title of the international time-scale). Although the overall mission of HMNAO had not really changed since its foundation, like superintendents Comrie and Sadler before him, George was always interested in pushing new methods of working to improve efficiency. He was a strong advocate of the use of SI units in astronomy.

On his retirement in 1989, just before the move to Cambridge, George and his wife Betty moved to Sidford on the outskirts of Sidmouth in Devon. He remained active within the IAU (Commission 4: Ephemerides; Commission 5: Documentation and Astronomical Data) for more than a decade. There was a very lively local astronomical society whose members became responsible for the Norman Lockyer Observatory when it re-opened in 1989 after many years of neglect. Lockyer (1836–1920), celebrated for his spectroscopic studies of stars, and famously the discoverer of the element helium in the solar spectrum, set up the NLO when he retired to Devon in 1912. Ex-RGO engineer John Pope, already a member, encouraged George to join and, while he was reluctant to get too deeply involved in administrative matters, he enthusiastically took part in the public outreach activities such as operating the on-site planetarium for the general public and organized parties, and giving talks. He also helped by making a successful grant application for a CCD system for securing improved images at the telescope, and completed the compilation of a manual for using the telescopes. From 1999 he became particularly interested in the history of the NLO and was still running the historical section until shortly before his death.

By any standards George lived a full and productive life and, although in his last years in Sussex he was deeply troubled by the move of the RGO to Cambridge and its subsequent demise eight years later; he was in 2009 able to look back and reflect: "I can only take some comfort from the continuance, in spite of the closure, of three activities for which I had been responsible in the 1980s, namely: H. M. Nautical Almanac Office; the Satellite Laser Ranging Group; and the archives of the RGO" — all of which are secure and thriving: a fitting legacy.

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Here and There

WATER WORLDS OR SAUNA BATHS?

… the larger [planets] are probably water worlds, although often with surface temperatures of 200 to 500C. — *The Daily Telegraph*, 2018 August 18, p. 9.

THE ANTHROPOID PRINCIPLE

A robust effective temperature–radius relationship for early man sequence. — RAS Meeting Notes, 2018 October 12.